Answers to Math 124 Winter 2022 Final

1. (a) Using L'Hospital's rule or factoring and canceling terms, the limit is 3. (b)-1.5 (c) e^{12}

2. (a) $\dfrac{dy}{dx} = \dfrac{\left(\frac{1}{3}x^{-2/3} + \cos x\right)(e^x+1) - \left(x^{1/3} + \sin x\right)e^x}{(e^x+1)^2}$

   (b) $\dfrac{dy}{dx} = \dfrac{\cos(2^x) \cdot \ln 2 \cdot 2^x}{\sin(2^x)}$

   (c) $\dfrac{dy}{dx} = \sec^2(x^x) \cdot (\ln x + 1) \cdot x^x + \dfrac{2x}{1+x^4}$

3. (a) $(-3, 2)$ and $(4, \infty)$ (b) -3,4 (c) $(-1.4, 3)$ and $(8, 10)$ (d) $-1.4, 3, 10$ (e) 18

4. (a) $\tan\theta = \dfrac{6-x}{x}$.

   (b) $x = 3$

   (c) Using $x - 3 \approx -3\left(\theta - \dfrac{\pi}{4}\right)$, $x - 3 \approx -\frac{3\pi}{64}$.

5. (a) $\dfrac{dy}{dx} = \dfrac{\pi}{2}t\sin\left(\dfrac{\pi}{2}t\right)$. When $t = 1$, the value is $\pi/2 > 0$ so y increases when x does.

   (b) $\dfrac{d^2y}{dx^2} = \dfrac{\pi}{2}t\left(\sin\left(\dfrac{\pi}{2}t\right) + \dfrac{\pi}{2}\cos\left(\dfrac{\pi}{2}t\right)\right)$. When $t = 1$, the value is $\pi/2 > 0$ so the graph is concave up.

   (c) B

6. $-\dfrac{29.68}{\sqrt{558.25}} \approx -1.256$ feet per second.

7. The radius is $\dfrac{5}{4+\pi}$, the width is $\dfrac{10}{4+\pi}$, and the height of the rectangular part is 2.5 (making the total height $2.5 + \dfrac{5}{4+\pi}$)

8. (a) $(1, 0)$ and $(0, 1)$.

   (b) $y = 0$ on the left.

   (c) $f'(x) = -xe^x$ so the only critical number is $x = 0$. Increasing on $(\infty, 0)$ and decreasing on $(0, \infty)$.

   (d) Local maximum at $(0, 1)$.

   (e) $f''(x) = -(x+1)e^x$ so concave up on $(-\infty, -1)$, concave down on $(-1, \infty)$.