**Math 124** **Final Examination** **Winter 2022**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

**READ THE INSTRUCTIONS!**

- Write you name, student number and quiz section clearly.
- Turn off and stow away all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied or printed materials are allowed.
- Give your answers in exact form. For example, $\frac{\pi}{3}$ or $5\sqrt{3}$ are exact numbers while 1.047 and 8.66 are decimal approximations for the same numbers.
- You can only use a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- Raise your hand if you have a question.
- This exam has 9 pages plus this cover page with 8 questions. Please make sure that your exam is complete.

1. (12 total points) Answer the following.

   (a) (4 points) Evaluate

   $$\lim_{x\to 0} \frac{e^{2x} + 4e^x - 5}{e^{2x} - 1}$$

   using *two different methods*, neither of which is guessing the answer from a table of values.

   *Using first method:*

   *Using second method:*

   (b) (4 points) Evaluate $\lim_{x\to\infty} \sqrt{4x^2 - 6x} - 2x$.

   (c) (4 points) Evaluate $\lim_{x\to 0^+} (1 + 4x)^{3/x}$.

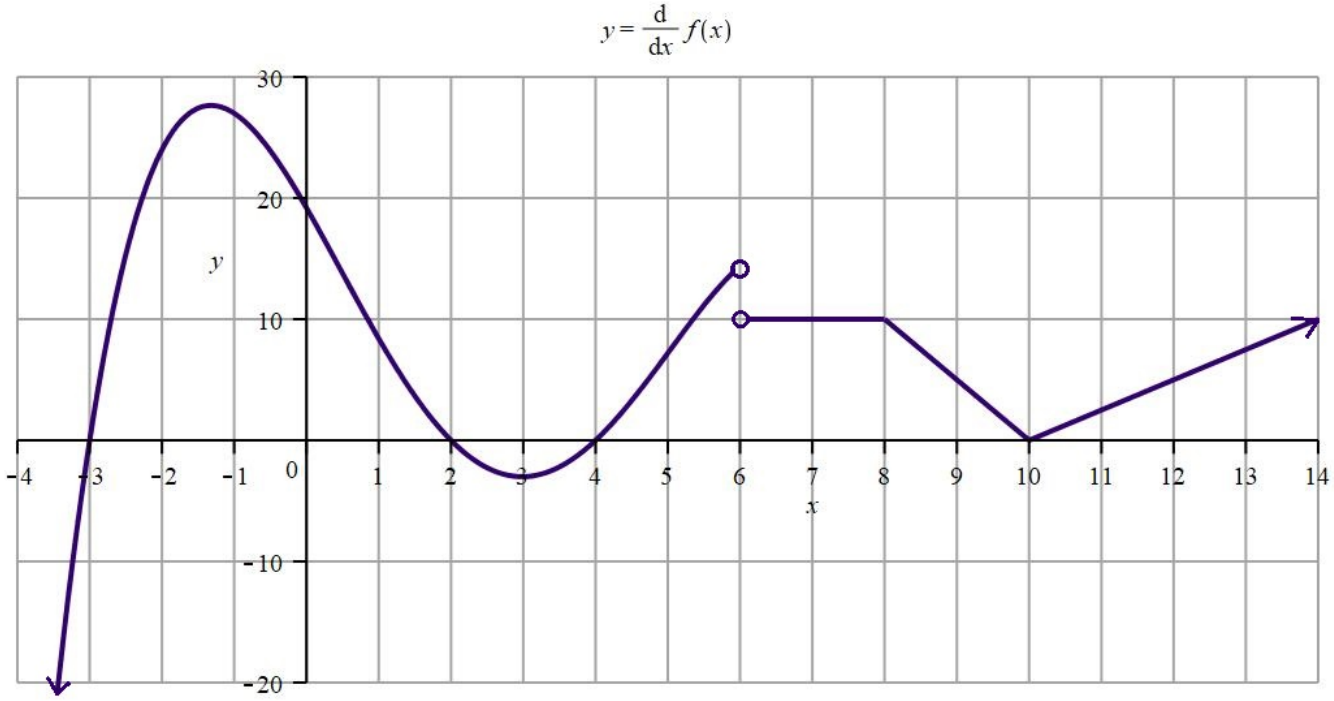
2. (13 total points) Find $\dfrac{dy}{dx}$. You do not need to simplify.

   (a) (4 points) $y = \dfrac{\sqrt[3]{x} + \sin(x)}{e^x + 1}$

   (b) (4 points) $y = \ln\left(\sin(2^x)\right)$

   (c) (5 points) $y = \tan(x^x) + \arctan(x^2)$

3. (10 points) The function $f(x)$ is continuous on its domain of all real numbers. The graph of its *derivative* is shown below. As you can see the derivative exists everywhere except at $x = 6$. Answer the questions based on the graph below. Give your best approximation if you are not sure about the exact numbers.




(a) List the intervals where the graph of $y = f(x)$ is increasing.

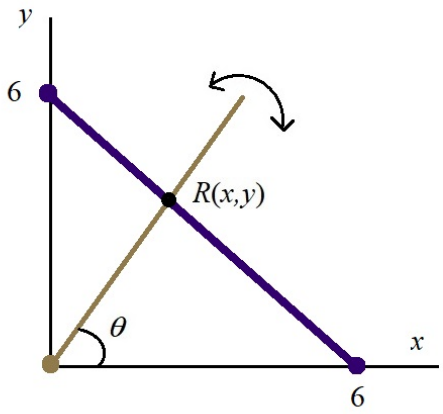
(b) List the $x$-values where the graph of $y = f(x)$ has a local minimum.

(c) List the intervals where the graph of $y = f(x)$ is concave down.

(d) List the $x$-values where the graph of $y = f(x)$ has an inflection point.

(e) If $f(6) = 3$, what is the value of $f(7.5)$?

4. (12 points) A metal rod is bolted on the $x$ and the $y$-axes at the points $(6, 0)$ and $(0, 6)$. A second rod is hinged at the origin and is free to rotate about the origin as shown. It intersects the first fixed rod at the point $R(x, y)$, which varies as the angle $\theta$ changes.




(a) Find an equation relating the angle $\theta$ shown with the $x$-coordinate of the point $R$.
*Hint: Relate $x$, $y$, and $\theta$ first and then try to eliminate the $y$ from the equation.*

(b) Use your equation to find the value of $x$ when $\theta = \dfrac{\pi}{4}$.

(c) Use *linear approximation* to approximate how much $x$ decreases if we increase $\theta$ from $\dfrac{\pi}{4}$ to $\dfrac{17\pi}{64}$ radians.

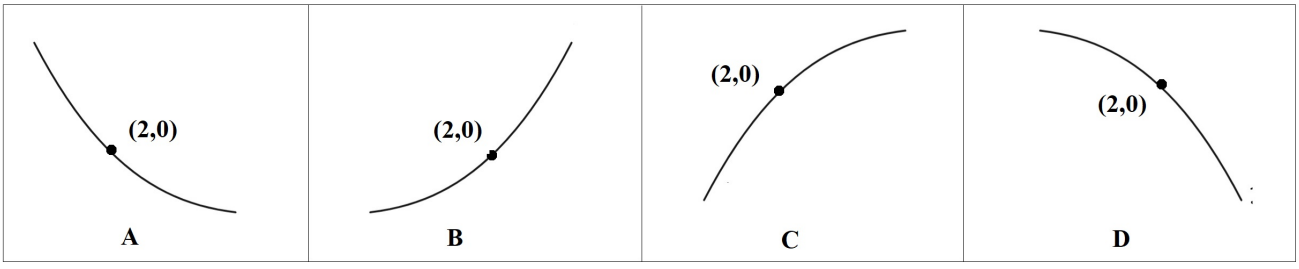
5. (12 points) The point $(2, 0)$ is on the curve defined by the parametric equations

$$x(t) = \ln(t) + 2 \quad \text{and} \quad y(t) = -\cos\left(\frac{\pi}{2}t\right).$$

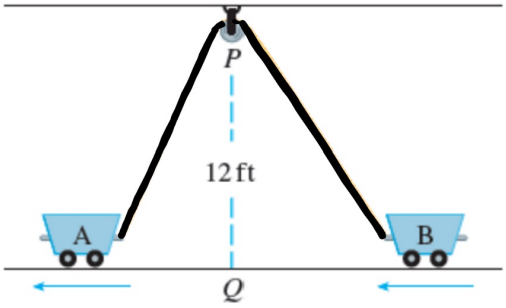
(a) Compute $\dfrac{dy}{dx}$ and determine if $y$ increasing when $x$ is increasing at $(2, 0)$?

(b) Using the second derivative, determine if the curve is concave up or concave down at $(2, 0)$.

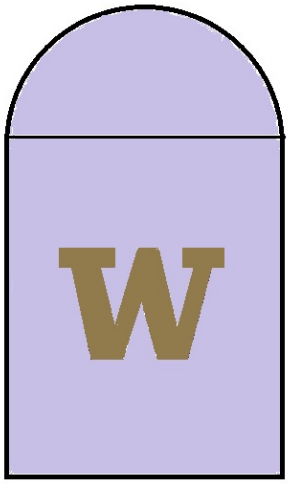
(c) Which of the options A, B, C, or D best describes the curve locally at $(2, 0)$? Justify your answer.



6. (12 points) Two carts, Cart $A$ and Cart $B$, are connected by a rope 39 feet long that passes over a pulley $P$. The point $Q$ is on the floor 12 feet directly beneath $P$ and between the carts. Cart $A$ is being pulled away from $Q$ at a speed of 4 feet per second. At what rate is the distance between Cart $B$ and the point $Q$ changing at the instance when Cart A is 3.5 feet from the point $Q$?



7. (11 points) A window has the shape of a half disk of radius $r$ placed on a rectangle as shown on the right. The perimeter of the window is to be 5 meters. What are the dimensions of the window which will maximize its area?
   Make sure you verify that you have indeed a maximum area.



8. (18 total points) Consider the function $f(x) = (1 - x)e^x$.

   (a) What are the $x$- and $y$-intercepts of the graph of $f$?

   (b) Determine the horizontal asymptotes of $f$.

   (c) Find all critical numbers of $f(x)$ and determine the intervals in which $f$ is increasing and in which it is decreasing.

   (d) Determine the local minima and local maxima of $f$.

*Continued on the next page.*

*Recall that the function is* $f(x) = (1-x)e^x$.

(e) Using $f''(x)$, find the intervals where $f$ is concave up and where it is concave down.

(f) Using **all** of the above information, sketch the graph of $f$ in the provided coordinate system.