Math 124 **Final Examination** **Spring 2024**

Honor Statement

I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.

Your Name

Your Signature

Student ID #

Quiz Section

Prof. Natalie Naehrig

READ THE INSTRUCTIONS!

- Silence your phone and put it away.
- You have 170 minutes for 8 problems. Check your copy of the exam for completeness.
- You are allowed to use a hand written sheet of paper (8x11 in), back and front.
- Calculator : TI 30 XIIS.
- Justify all your answers and show your work for credit.
- All answers must be exact, no rounding, unless otherwise indicated.
- The total of this exam is 100 points.
- The last page is for scratch paperwork and will not be graded unless you indicate so.

Do not open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!

1. (12 points) Determine the limits. Your answer should be a number (**no rounding**), ∞, $-\infty$ or DNE, whatever fits best. Put a box around your final answers.

   (a) $\lim_{x\to-\infty} \dfrac{2x^3-4x^2+6x+8}{8x^3-6x^2+8x+1}$

   (b) $\lim_{x\to 1} \dfrac{x^2-1}{\sqrt{x^2+3}-2}$

   (c) $\lim_{x\to 0^+} \arctan(\ln(x))$

   (d) $\lim_{x\to 1} \left(\dfrac{e^{x-1}-1}{x-1}\right)$

2. (12 points) Find the derivative of the following functions. Do **not** simplify!

   Put a box around your final answers.

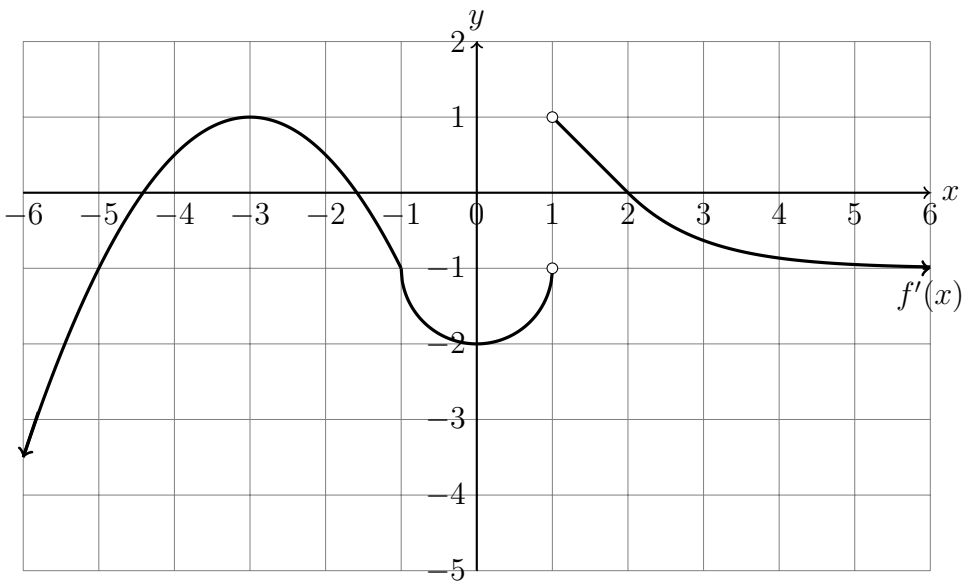
   (a) $f(x) = \dfrac{\ln(3x)}{xe^x}$

   (b) $g(x) = \sqrt{\sin(x^2 + \sqrt{x})}$

   (c) $h(x) = x^{2x} + x^2 + 2^x$

   (d) $k(x) = \arctan(4x^2)$

3. (10 points) For this problem, you do not need to show your work. Answer the questions based on the following graph of the **derivative of a function** $f(x)$. The domain of the $f(x)$ is $(-\infty, \infty)$.



(a) $f'(-5) =$

(b) $f'(-1) =$

(c) $\lim_{x \to -3} f'(x) =$

(d) $\lim_{x \to \infty} f(x) =$

(e) $\lim_{x \to \infty} f'(x) =$

(f) List all x-values where f is not differentiable.

_____

(g) List all critical values of f.

_____

(h) List all x-values where f has a local maximum.

_____

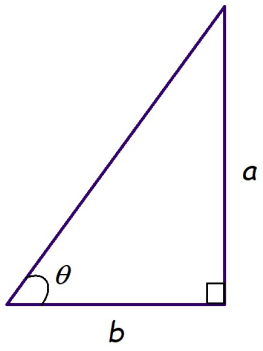
(i) Find the x-value in $[2, 5]$ where f has an absolute minimum.

_____

(j) List all intervals where f is concave up.

_____

4. (12 points) Consider a right triangle in which the lengths of the two legs a and b, as shown in the sketch, change. The length of the leg opposite the angle θ is 110cm at the beginning and decreases by 5cm per minute in length. The length of the adjacent leg of θ is 10cm at the beginning and increases by 15cm per minute in length. How does the angle θ change when the triangle is isosceles? Do not round.

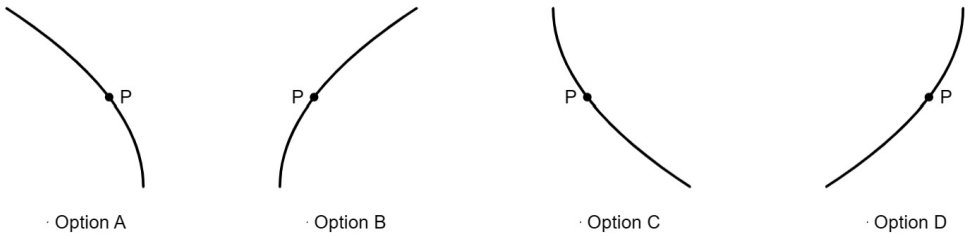


5. (12 points) You are tasked to build a box of metal with a square base and an open top whose volume is 4m^3. Find the dimensions of the box in meters that minimize the amount of material used. Justify that your answer is indeed an absolute minimum.

6. (12 points) Consider the curve defined by the parametric equations $x(t) = \cos(t)$ and $y(t) = \sin(2t)$, where $0 \leq t \leq 2\pi$.

   (a) What points on the curve have a horizontal tangent line?

   (b) The time $t = \pi/3$ gives rise to a point P on the curve. Which of the following options resembles the given curve around P most?



Option A Option B Option C Option D

7. (12 points) Consider the implicitly defined curve $\ln(y) + y + xy^2 = 5$ and the point $(4, 1)$ on the curve.

   (a) Through linearization find an approximation of the y-coordinate of point $(4.1, y)$ on the curve.

   (b) Is it an over- or under-estimate? Justify your answer.

8. (18 points) Consider the function $f(x) = \dfrac{x}{x^2+1}$.

   (a) Find the y-intercept of the function.

   (b) Find the asymptote(s) of the function if any.

   (c) Find all critical numbers of $f(x)$

   (d) List all intervals where f is increasing and all intervals where f is decreasing.

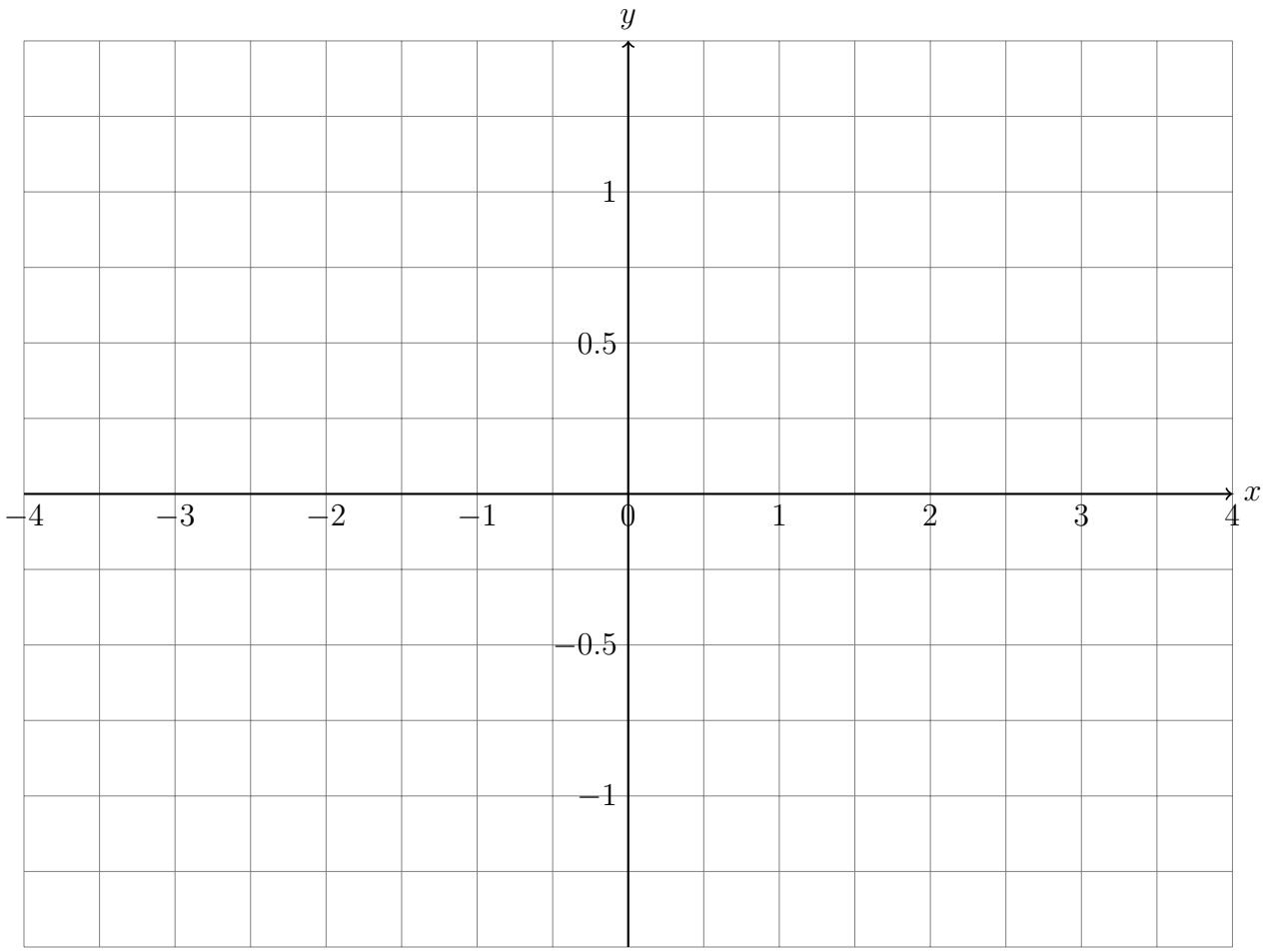
Recall that the function is: $f(x) = \dfrac{x}{x^2+1}$.

(e) Determine all points $P(x, y)$ (x and y-coordinates!) where f has a local maximum or a local minumum.

(f) List all intervals, where f is concave up and all intervals where f is concave down.

(g) Determine all points $Q(x, y)$ that are inflection points of f.

(h) Graph the function in the given coordinate system. All results (a)-(g) must be used in your graph.



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