Math 124 **Final Examination** **Spring 2023**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

READ THE INSTRUCTIONS!

- *These exams will be scanned. Write your name, student number and quiz section clearly.*
- Turn off and stow away all cell phones, smart watches, mp3 players, and other similar devices. No earbuds/headphones allowed during the exam.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied or printed materials are allowed.
- Give your answers in exact form unless instructed otherwise. For example, $\frac{\pi}{3}$ or $5\sqrt{3}$ are exact numbers while 1.047 and 8.66 are decimal approximations for the same numbers.
- You can only use a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- This exam has 11 pages plus this cover page with 8 questions. Please make sure that your exam is complete.

Problem	Score	Problem	Score	Problem	Score
1 (14 pts)		4 (12 pts)		7 (13 pts)	
2 (12 pts)		5 (12 pts)		8 (15 pts)	
3 (10 pts)		6 (12 pts)		**Total**	

1. (14 total points) Answer the following.

   (a) (5 points) Compute $\lim_{x\to 0^+} (x \ln x)$ and $\lim_{x\to 0^+} \left(\frac{1}{x} + \frac{1}{\ln x}\right)$

   (b) (4 points) Leibniz computed $\lim_{x\to 2} \frac{x^2-4}{9-(f(x))^2} = 5$ using L'Hospital's Rule. What is $f(2)$? What is $f'(2)$?

   $f(2) =$ $\square$ $\quad$ $f'(2) =$ $\square$

   (c) (5 points) Determine if the following are True or False. Circle the correct answer.

   True False (i) If $\lim_{x\to 3} f(x) = 5$ and $\lim_{x\to 3} g(x) = 0$, then $\lim_{x\to 3} \frac{f(x)}{g(x)} = \infty$.

   True False (ii) If $\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$, then $\lim_{x\to a} f'(x) = \lim_{x\to a} g'(x)$.

   True False (iii) If $f'(x) = g'(x)$ for all x, then $f(x) = g(x)$ for all x.

   True False (iv) If $\lim_{x\to 4^+} f(x) = \infty$, then $x = 4$ is a vertical asymptote for the graph $y = f(x)$.

   True False (v) If $f'(x) = \frac{3x^2+e^x}{\cos x}$, then $f(x) = \frac{x^3+e^x}{\sin x}$.

2. (12 total points) Find the derivatives of the following functions. You do not have to simplify your answers.

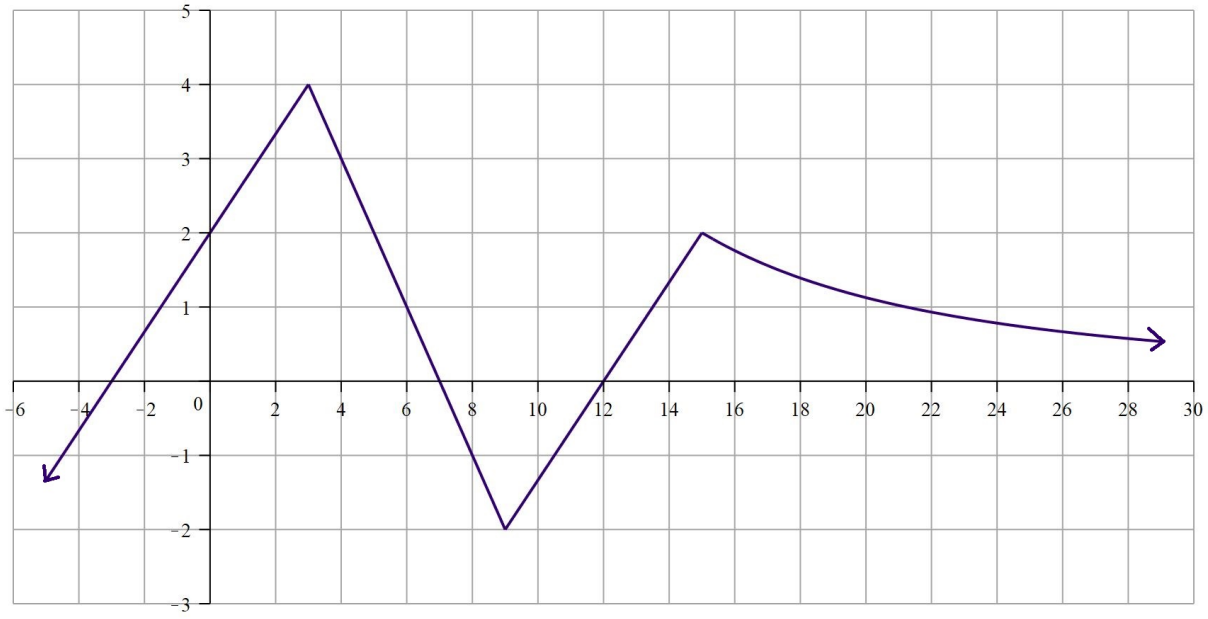
   (a) (4 points) $h(x) = \sqrt{2\sin^2(x) + 1}$.

   (b) (4 points) $g(x) = (ax^2 + b)e^{-cx}$ where a, b, c are constants.

   (c) (4 points) $y = (\cos(x))^{x^2}$.

3. (10 points) The function $f(x)$ is differentiable everywhere and $f(0) = 0$. Answer the questions based on the graph of $y = f'(x)$, **the derivative of** $f(x)$, shown below.

$$y = f'(x)$$



(a) $\lim_{x \to 0} \dfrac{f(x)}{x} =$ ☐

(b) $f''(11) =$ ☐

(c) List all values of x where the graph of $y = f(x)$ has a local minimum.

Answer: ☐

(d) List all intervals where the graph of $y = f(x)$ is concave down.

Answer: ☐

(e) If $g(x) = f\left(f(x)\right)$, what is $g'(0)$?

$g'(0) =$ ☐

4. (12 total points) One point on the curve defined by

$$x^4 + 3xy + y^4 = 5$$

is $P = (1, 1)$.

(a) (4 points) Find the formula for the implicit derivative $\dfrac{dy}{dx}$.

(b) (4 points) Write the equation of the tangent line at P.

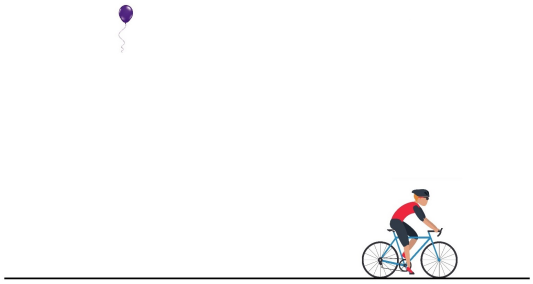
(c) (4 points) Is the curve defined by the equation concave up or concave down at the point P? Explain.

5. (12 points) A pizza delivery chain has found that the number of veggy pizzas it can sell on a Friday night is closely modeled by the function

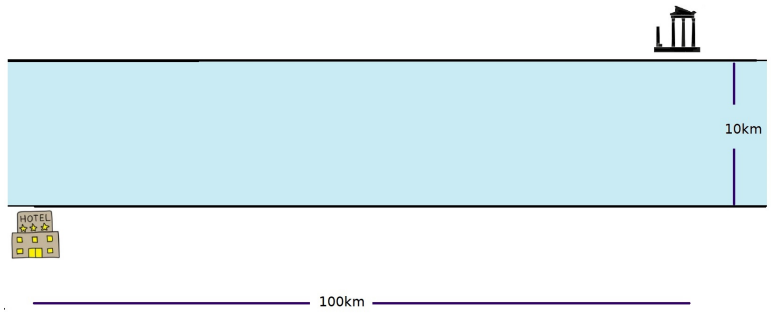
$$N(x) = 1000 - 240\ln(x/12) - 20x,$$

where x is the cost of the pizza in dollars. The current price is \$12. The pizza chain want to have their sales on Friday night increase by 50 pizzas. Use linear approximation to estimate the required new pizza price.

6. (12 points) A balloon is at a height of 40 meters, and is rising at the constant rate of 8m/sec. At that instant a bicycle passes beneath it, traveling in a straight line at the constant speed of 10m/sec. How fast is the distance between them increasing 3 seconds later?
The picture below is not to scale.



7. (13 points) A tourist group is going to see the ruins across the channel of water. The ruins are at a 100 kilometer horizontal distance from the hotel at a point 10 kilometers across a body of water as shown on the diagram. They can take a bus to drive them by the shore and then they can get on a boat at any point on the shore to get to the other side. The bus travels at 45 kilometers per hour and the boat at 30 kilometers per hour. What is the shortest amount of time in which they can make it to the ruins?
Assume that there is no current and the boat can go straight to the ruins. The drawing is not to scale.



8. (15 points) Let $f(x) = 0.5x^2e^{-0.1x}$ be defined on the domain $D = [-5, 50]$.

   (a) Determine the sub-intervals of D where $f(x)$ is increasing.

   (b) Determine the sub-intervals of D where the graph of $y = f(x)$ is concave up.

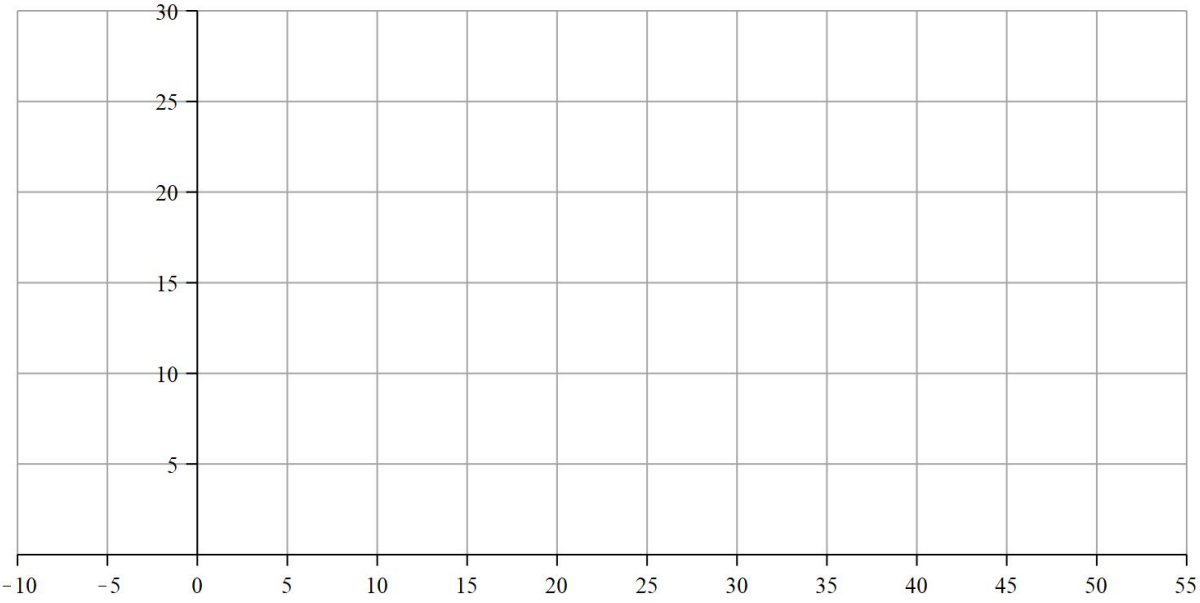
Continued on the next page.

8. (continued) Recall the function $f(x) = 0.5x^2e^{-0.1x}$

   (c) Find the inflection points on the graph, if any. *You can give your answer in decimals rounded to one digit after the decimal.*

   (d) Find the absolute maximum and absolute minimum values of the function on D. *You can give your answer in decimals rounded to one digit after the decimal.*

   (e) Sketch the graph $y = f(x)$ using the grid below. Clearly label the (x, y) coordinates of endpoints, all critical points, and points of inflection. Make sure your graph matches with the information you provided above.



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