Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

READ THE INSTRUCTIONS!

- *These exams will be scanned. Write you name, student number and quiz section clearly.*
- Turn off and stow away all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied or printed materials are allowed.
- Give your answers in exact form. For example, $\frac{\pi}{3}$ or $5\sqrt{3}$ are exact numbers while 1.047 and 8.66 are decimal approximations for the same numbers.
- You can only use a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- This exam has 9 pages plus this cover page with 8 questions. Please make sure that your exam is complete.

Problem	Score	Problem	Score	Problem	Score
1 (15 pts)		4 (12 pts)		7 (11 pts)	
2 (18 pts)		5 (12 pts)		8 (15 pts)	
3 (12 pts)		6 (12 pts)		**Total**	

1. (15 total points) Answer the following.

   (a) (6 points) Evaluate

$$\lim_{\theta \to \pi/2} \frac{\sin^2\theta - 5\sin\theta + 4}{\sin^2\theta - 1}$$

   using *two different methods*, neither of which is guessing the answer from a table of values.

   Using first method:

   Using second method:

   (b) (4 points) Evaluate $\lim_{x \to 3} \left(\frac{1}{\sqrt{x+1}} - \frac{1}{2} \right)$.

   (c) (5 points) Find the values of a and c so that $f(x)$ is continuous $x = 0$.

$$f(x) = \begin{cases} \frac{e^{ax} - 1}{x} & x > 0 \\ c & x = 0 \\ \frac{\sin(3x)}{x} & x < 0 \end{cases}$$

2. (18 total points) Find the derivatives of the following functions. You do not have to simplify.

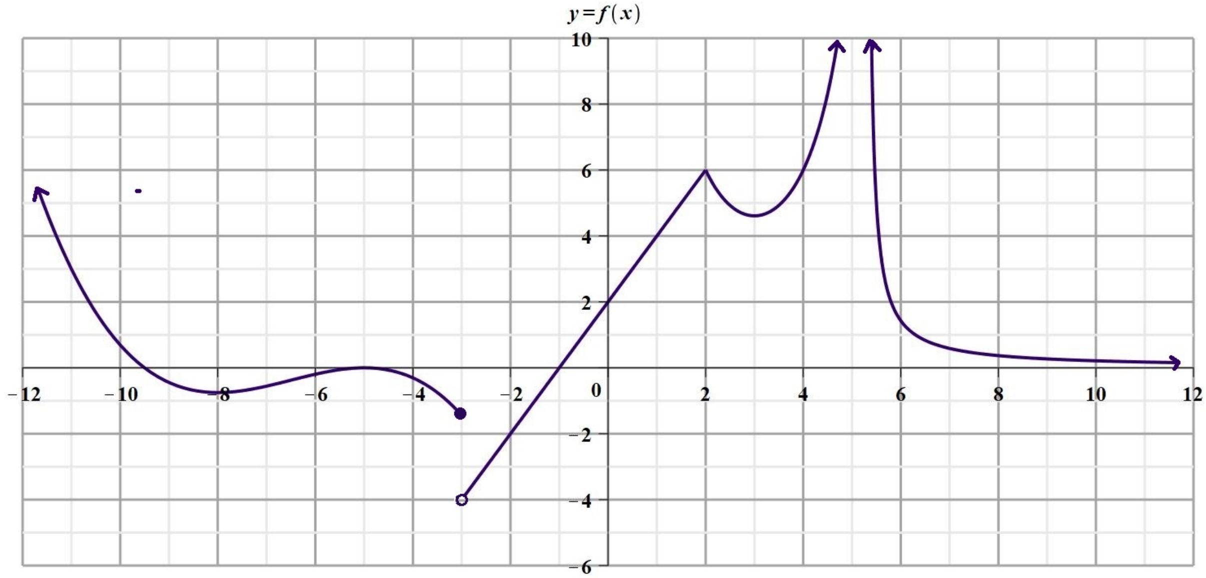
   (a) (4 points) $f(x) = 3^{\tan(4x)}$

   (b) (4 points) $h(x) = \sin(x^2)\cos^2(3x)$

   (c) (5 points) $y(x) = x^{e^x}$

   (d) (5 points) $g(x) = \dfrac{4xe^x}{\sqrt{x^4+5}}$

3. (12 points) The function $f(x)$ whose graph is given below has domain all numbers except for $x = 5$. Answer the questions based on the graph below.



(a) $\lim_{x \to 5} f(x) =$

(b) $\lim_{x \to -3^+} f(x) =$

(c) $\lim_{h \to 0} \dfrac{f(h) - f(0)}{h} =$

(d) Give your best estimate for $f(1.85)$.

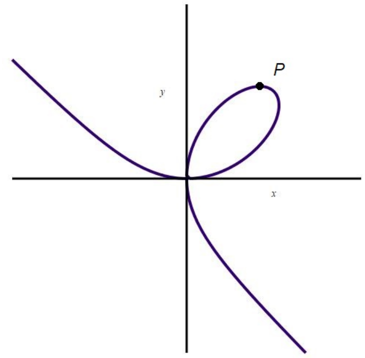
(e) Let $g(x) = (f(x))^2$. Find all values of x where $g'(x) = 0$.

(f) Let $h(x) = f(f(x))$. Compute $h'(3)$.

4. (12 points) A curve is given implicitly by the equation

$$x^3 + y^3 = 2xy.$$

A graph is shown on the right to help you visualize.



(a) Find the equation of the tangent line to the curve at the point $(1, 1)$.

(b) Use *linear approximation* to find the value of x when $y = 0.93$.

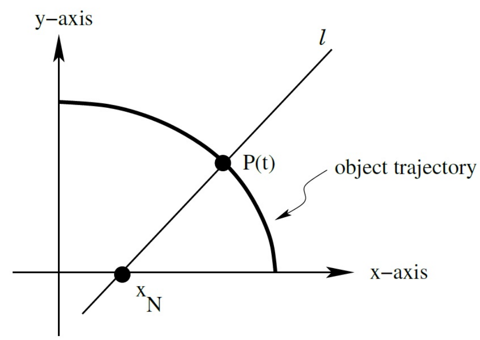
(c) Find the coordinates of the point P on the curve where the tangent line is horizontal.

5. (12 points) An object is moving in the xy-plane according to the parametric equations:

$$\begin{aligned} x(t) &= 5\cos(\pi t) \\ y(t) &= 4\sin(\pi t) \end{aligned}$$

When $0 < t < \frac{1}{2}$, the location $P(t) = (x(t), y(t))$ of the object will be in the first quadrant, as pictured below. Let ℓ be the normal line to the trajectory at $P(t)$ and x_N the x-intercept of ℓ. (The normal line ℓ is perpendicular to the tangent line through $P(t)$.)

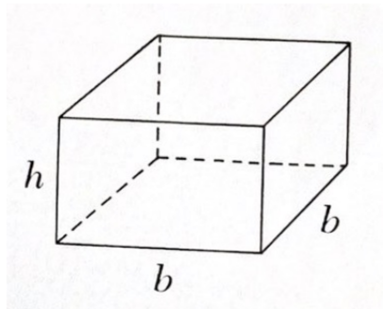
(a) Write the equation of the normal line ℓ through $P(t)$, assuming $0 < t < \frac{1}{2}$.



(b) Find an expression for x_N as a function of t.

(c) Compute $\lim\limits_{t \to 0^+} x_N =$

6. (10 points) The dimensions of a rectangular box with square bottom are changing. The length of the side of the base is increasing at a constant rate of 3 ft/min and the height is decreasing at a constant rate of 5 ft/min.



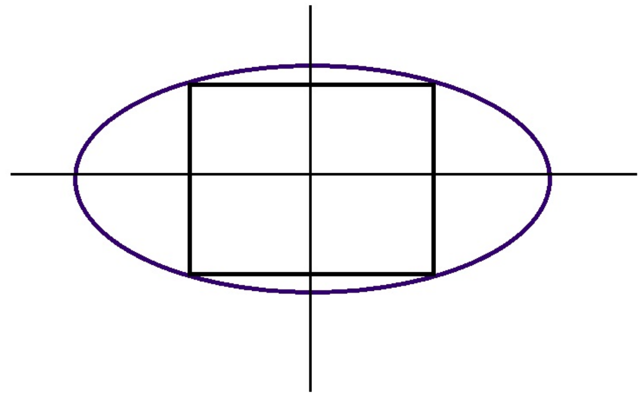
(a) What is the rate of change of the volume of the box when the length of the side of the base is 4 ft and the height is 3 ft?

(b) Is the rate of change of the volume of the box increasing or decreasing when the length of the side of the base is 4 ft and the height is 3 ft? Explain.

7. (11 points) A rectangle is inscribed in the ellipse

$$\frac{x^2}{400} + \frac{y^2}{225} = 1$$

with sides parallel to the axes, as pictured. Find the dimensions of the rectangle of maximum perimeter which can be so inscribed.



8. (15 total points) Consider the function $f(x) = \dfrac{8}{x^2+4}$.

   (a) What are the x- and y-intercepts of the graph of f?

   (b) Determine the horizontal asymptotes of f.

   (c) Find all critical numbers of $f(x)$ and determine the intervals in which f is increasing and in which it is decreasing.

   (d) Determine the local minima and local maxima of f.

Continued on the next page.

Recall that the function is $f(x) = \dfrac{8}{x^2+4}$.

(e) Using $f''(x)$, find the intervals where f is concave up and where it is concave down.

(f) List the inflection points on the graph of f.

(g) Using **all** of the above information, sketch the graph of f in the provided coordinate system. Mark any important points that came up in your computations.