Math 124 **Final Examination** **Autumn 2024**

Your Name

Your Signature

Student ID #

Quiz Section

Put a check next to your professor's name:

December 7, 2024

- ☐ Prof. David Collingwood
- ☐ Prof. Fanny Dos Reis
- ☐ Prof. Patrick Perkins
- ☐ Prof. Danny Shi

READ THE INSTRUCTIONS!

- *These exams will be scanned. Write your name, student number and quiz section clearly.*
- Turn off and stow away all cell phones, smart watches, and other similar devices. No earbuds, headphones, or any kind of connected devices allowed during the exam.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied or printed materials are allowed.
- Give your answers in exact form unless instructed otherwise. For example, $\frac{\pi}{3}$ or $5\sqrt{3}$ are exact numbers while 1.047 and 8.66 are decimal approximations for the same numbers.
- You can only use a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- This exam has 11 pages plus this cover page with 8 questions. Please make sure that your exam is complete.

Problem	Score	Problem	Score	Problem	Score
1 (12 pts)		4 (12 pts)		7 (13 pts)	
2 (12 pts)		5 (13 pts)		8 (18 pts)	
3 (8 pts)		6 (12 pts)		**Total**	

1. (12 total points) Compute each of the following limits showing complete work or justification for your answer. If there is no finite limit, write ∞, $-\infty$, or DNE, whichever applies.

   (a) (4 points) $\lim_{x\to 2} \dfrac{\sqrt{4x+1}-3}{x-2}$

   (b) (4 points) $\lim_{\theta\to 0} \dfrac{\sin 3\theta}{\tan 2\theta}$

   (c) (4 points) $\lim_{x\to\infty} x^{\frac{3}{2}} \sin\left(\dfrac{1}{x}\right)$

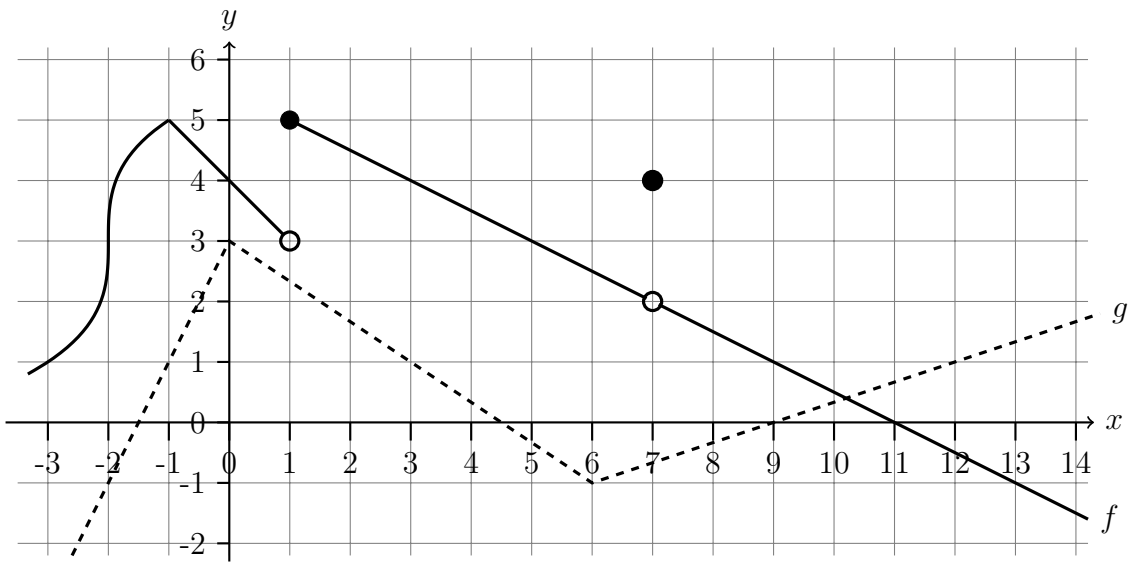
2. (12 total points) Find the derivatives of the following functions. Do not simplify your answers.

   (a) (4 points) $f(x) = \sin^2(\pi x)\cos(\pi^2 x)$.

   (b) (4 points) $f(x) = x^2\sqrt{1+\sqrt{2x}}$

   (c) (4 points) $f(x) = (1+\ln x)^{\ln x}$

3. (8 points) The questions below are based on the graphs of $y = f(x)$ (solid line) and $y = g(x)$ (dashed line) below.



(a) List all values of x where the function f is not differentiable.

(b) List all the values of x in the interval $[5, 14]$ where the function $k(x) = g(f(x))$ is not differentiable.

(c) $g'(12) =$ ☐

(d) $\lim_{x \to 3^+} (f(g(x))) =$ ☐

(e) If $h(x) = \dfrac{g(x)}{f(x)}$, then

$h'(3) =$ ☐

$\lim_{x \to 11^+} h(x) =$ ☐

(f) $\lim_{x \to 9} \dfrac{g(x)}{f(x) - 1} =$ ☐

(g) If $k(x) = g(f(x))$,

then $k'(0) =$ ☐

4. (12 total points) A curve is given implicitly by the equation

$$2(x^2 + y^2)^2 = 25(x^2 - y^2).$$

(a) (7 points) Find the equation of the tangent line to the curve at the point $(3, 1)$.

(b) (5 points) Compute the value of y'' at the point $(3, 1)$.

5. (13 points) Consider the curve defined by the parametric equations

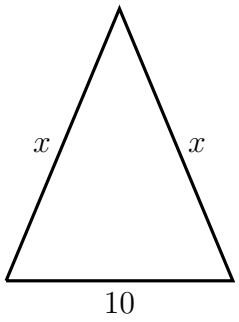
$$x(t) = 2t + \ln t \quad \text{and} \quad y(t) = t^3 - 2t, \quad \text{where} \quad t > 0.$$

Note that $(2, -1)$ is the point on the curve where $t = 1$.

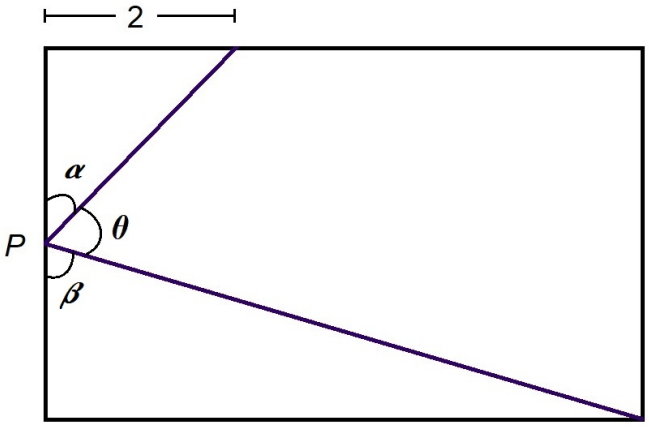
(a) Through linearization find an approximation of the y-coordinate of point $(1.9, y)$ on the curve.

(b) Is it an over- or under-estimate? Justify your answer.

6. (12 points) Consider the given isosceles triangle of base 10 inches and side lengths x inches. If x is increasing at the rate of 3 in/min., at what rate is the triangle's area increasing when $x = 13$ inches?



7. (13 points) Starting with a rectangle 4 inches high and 6 inches wide, construct the interior line segments as pictured. Find the location P so that the angle θ is as big as possible.
Hint: $\alpha + \theta + \beta = \pi$ radians.



8. (18 total points) Consider the function $f(x) = 3xe^{-x^2/6}$.

   (a) (1 point) Find the y-intercept of the function.

   (b) (2 points) Find the asymptote(s) of the function if any.

   (c) (4 points) Find all critical numbers of $f(x)$.

   (d) (2 points) List all intervals where f is increasing and all intervals where f is decreasing.

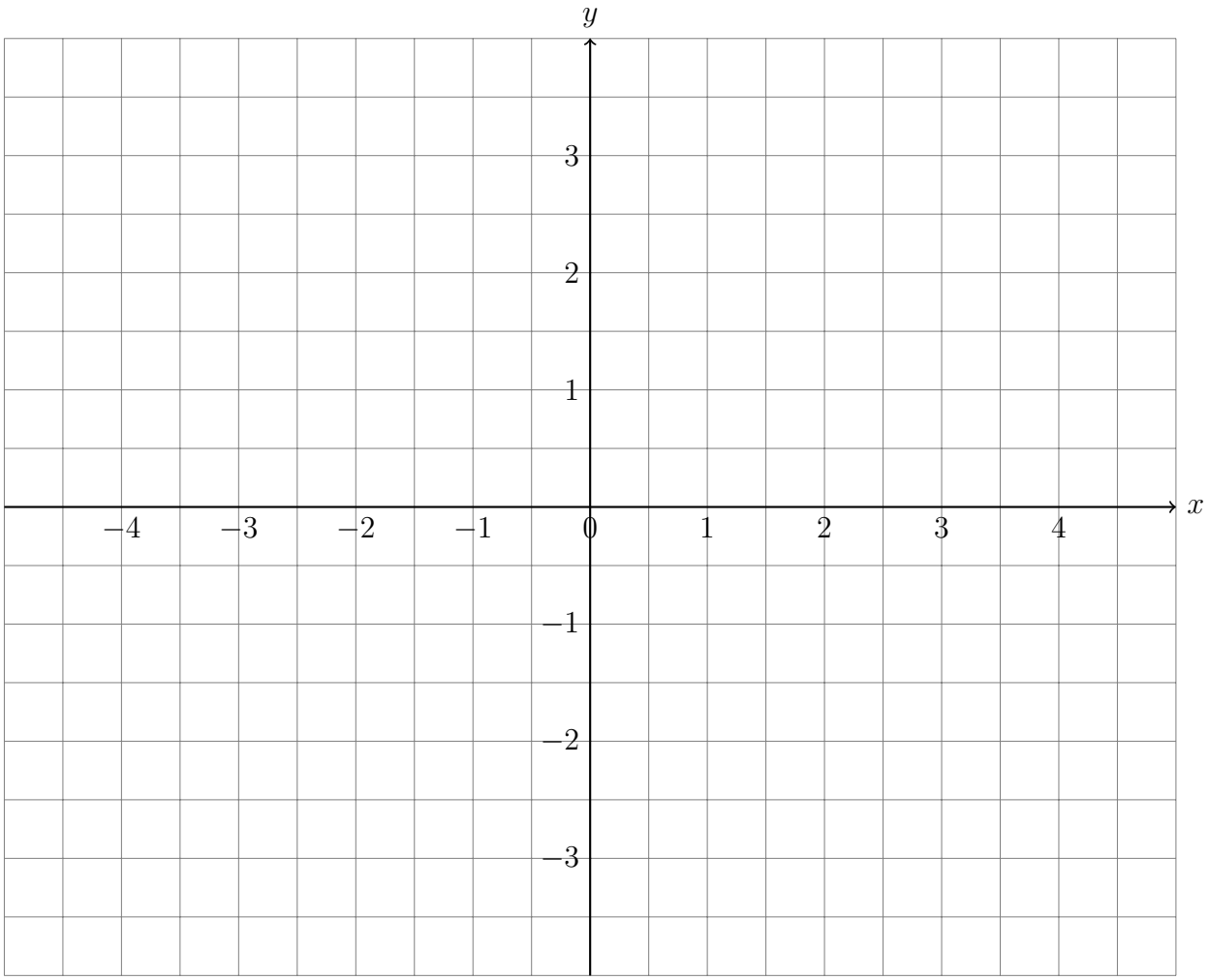
Recall that the function is: $f(x) = 3xe^{-x^2/6}$.

(e) (3 points) Determine all points (a, b) (both coordinates) where f has a local maximum or a local minumum.

(f) (4 points) List all intervals, where f is concave up and all intervals where f is concave down.

(g) (2 points) Determine all points (a, b) that are inflection points of f.

(h) Sketch the graph of $y = f(x)$ on the grid provided below. Be sure to include any asymptotes in your picture, and to mark the coordinates of all local maxima, local minima, and inflection points (if any exist).



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