Math 124 **Final Examination** **Autumn 2023**

Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

READ THE INSTRUCTIONS!

- *These exams will be scanned. Write your name, student number and quiz section clearly.*
- Turn off and stow away all cell phones, smart watches, and other similar devices. No earbuds, headphones, or any kind of connected devices allowed during the exam.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied or printed materials are allowed.
- Give your answers in exact form unless instructed otherwise. For example, $\frac{\pi}{3}$ or $5\sqrt{3}$ are exact numbers while 1.047 and 8.66 are decimal approximations for the same numbers.
- You can only use a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- This exam has 11 pages plus this cover page with 8 questions. Please make sure that your exam is complete.

Problem	Score	Problem	Score	Problem	Score
1 (12 pts)		4 (12 pts)		7 (13 pts)	
2 (15 pts)		5 (12 pts)		8 (14 pts)	
3 (10 pts)		6 (12 pts)		**Total**	

1. (12 total points) Compute each of the following limits showing complete work or justification for your answer. If there is no finite limit, write ∞, $-\infty$, or DNE, whichever applies.

   (a) (4 points) $\lim_{x\to 2} \dfrac{\sqrt{x^2+5}-3}{x^2-2x}$

   (b) (4 points) $\lim_{x\to \pi/2} \dfrac{1-\sin x}{x-\pi/2}$

   (c) (4 points) $\lim_{x\to 0^+} (\sin x)^{\tan x}$

2. (15 total points) Find the derivatives of the following functions. Do not simplify your answers.

   (a) (5 points) $f(x) = \dfrac{x\sin(x^3)}{1+e^{4x}}$.

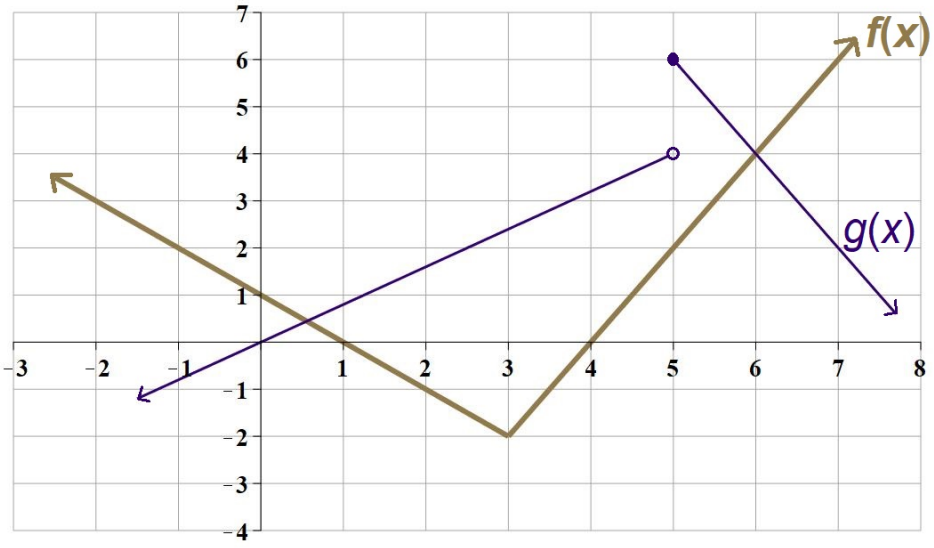
   (b) (5 points) $g(x) = \tan\left(a + \ln\left(b + e^{cx}\right)\right)$ where a, b, c are constants.

   (c) (5 points) $k(x) = \arctan\left(\dfrac{2x-1}{4+7x}\right)$.

3. (10 points) The following shows the graphs of $y = f(x)$ (thick lines) and $y = g(x)$ (thin lines). Both functions are defined for all real numbers, $f(x)$ is continuous, but $g(x)$ has a discontinuity at $x = 5$ with $g(5) = 6$. Let

$$u(x) = g\left(f(x)\right), \qquad v(x) = \left(f(x)\right)^2, \qquad w(x) = f(x)g(x).$$

Answer the following questions in the boxes provided. You do not have to show your work.



(a) $f'(0) =$ ☐

(b) $w'(7) =$ ☐

(c) $u'(0) =$ ☐

(d) List all values of x where $u(x)$ is not differentiable.

Answer: ☐

(e) List all intervals where the graph of $v(x)$ is increasing.

Answer: ☐

4. (12 points) A curve is given implicitly by the equation

$$2(x^2 + y^2)^2 = 25(x^2 - y^2).$$

Find the points on the curve with $x \neq 0$ where the tangent line is vertical.

5. (12 points) Consider the parametric curve defined by the following equations:

$$x(t) = e^t + t$$
$$y(t) = t^3 + 4t^2 + 1$$

for $-\infty < t < \infty$.

(a) (6 points) Find the values of t for which the tangent line to the curve is either horizontal or vertical, making sure you specify which values correspond to which type of tangent.

(b) (6 points) At the times t for which that tangent line is horizontal or vertical, determine whether the curve is concave up or concave down .

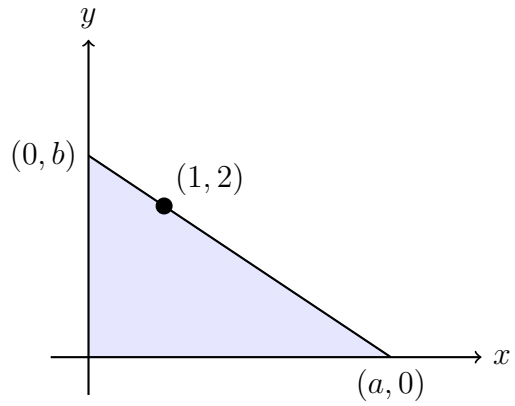
6. (12 points) You are located 5 miles directly south of an island when you see a ship heading away from the island, sailing due east. At the moment when you see the ship, the angle θ measured clockwise from directly north of you to the ship is $\frac{\pi}{6}$ radians, and it's increasing at a rate of $5/4$ radians per hour.

   What is the speed of the ship? Include units. *Figure below is not to scale.*

7. (13 points) A right triangle is formed in the first quadrant by the x and the y-axes, and by a line passing through the point with coordinates $(1, 2)$.

   Find the x and y-intercepts a and b of the line such that the area of the triangle formed is minimal.

   Check that the values you found correspond to a minimum.



8. (14 points) Consider the function

$$f(x) = \frac{e^x}{x}$$

.

(a) (2 points) Compute $\lim_{x \to 0^-} f(x)$ and $\lim_{x \to 0^+} f(x)$.

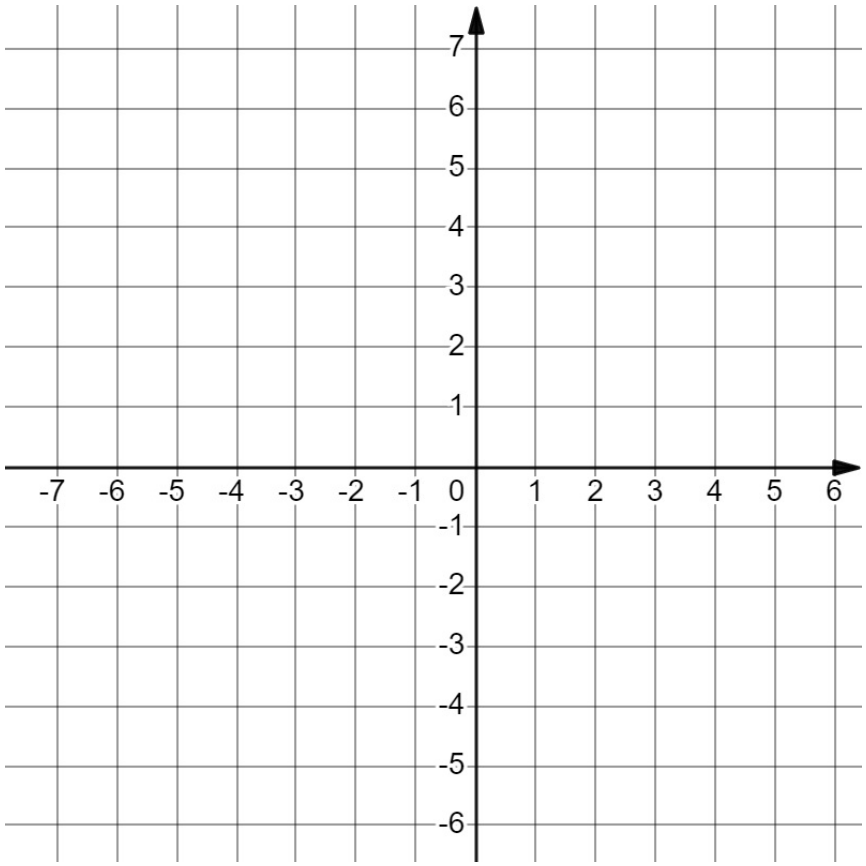
(b) (2 points) Find the horizontal asymptotes of $y = f(x)$, if there are any. Justify your answer via limit calculations.

(c) (3 points) Compute $f'(x)$. Determine all intervals where $y = f(x)$ is increasing or decreasing.

Recall that the function is: $f(x) = \dfrac{e^x}{x}$.

(d) (3 points) Determine all the intervals where the function is concave down or concave up.

(e) (4 points) Sketch the graph of $y = f(x)$ on the grid provided below. Be sure to include any asymptotes in your picture, and to mark the coordinates of all local maxima, local minima, and inflection points (if any exist).



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