Your Name

Your Signature

Student ID #

Quiz Section

Professor's Name

TA's Name

READ THE INSTRUCTIONS!

- *These exams will be scanned. Write you name, student number and quiz section clearly.*
- Turn off and stow away all cell phones, smart watches, mp3 players, and other similar devices. No earbuds/headphones allowed during the exam.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied or printed materials are allowed.
- Give your answers in exact form unless instructed otherwise. For example, $\frac{\pi}{3}$ or $5\sqrt{3}$ are exact numbers while 1.047 and 8.66 are decimal approximations for the same numbers.
- You can only use a Texas Instruments TI-30X IIS calculator.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- This exam has 11 pages plus this cover page with 8 questions. Please make sure that your exam is complete.

Problem	Score	Problem	Score	Problem	Score
1 (14 pts)		4 (12 pts)		7 (11 pts)	
2 (15 pts)		5 (11 pts)		8 (15 pts)	
3 (12 pts)		6 (10 pts)		**Total**	

1. (14 total points) Determine each of the following limits. If there is no finite limit, write ∞, $-\infty$, or DNE (does not exist), whichever applies.

   (a) (5 points) $\displaystyle\lim_{x\to 0} \frac{\cos(3x) - 1}{x^2}$

   (b) (4 points) $\displaystyle\lim_{x\to 0^+} \frac{e^x + e^{-x}}{e^x - e^{-x}}$

   (c) (5 points) $\displaystyle\lim_{x\to \infty} (\ln(2x + 1) - \ln(x + 1))$

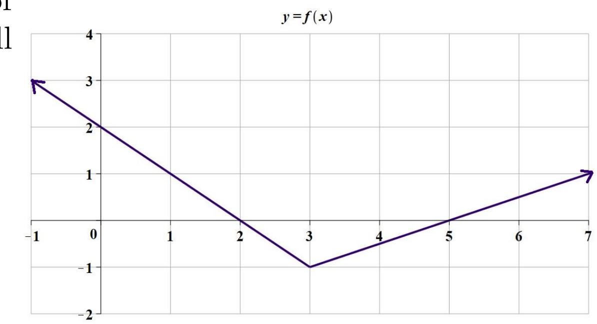
2. (15 total points) Find the derivatives of the following functions. You do not have to simplify.

   (a) (5 points) $f(x) = \dfrac{\arctan(2x) + x^2}{e^{3x}}$.

   (b) (5 points) $g(x) = \sin(x^4 + e^{x^2+x})$.

   (c) (5 points) $h(x) = \ln(x) \cdot \arctan(4x^2) \cdot \sqrt{x^2 + x}$.

3. (12 points) Answer the questions based on the graph of $f(x)$ given on the right. The domain of f is the set of all numbers.



Let $g(x) = f\left(f(x)\right)$ and let $h(x) = \left(f(x)\right)^2$.

(a) Compute $f'(2.5)$.

(b) Compute $g'(5)$.

(c) Find all critical numbers for $g(x)$. Is g differentiable everywhere?

(d) Compute $\displaystyle\lim_{x\to 0} \frac{g(x)}{x}$.

(e) Find the intervals where h is increasing.

4. (12 points) Consider the curve defined by the equation $y = x^{2y}$.

   (a) Confirm that $(1, 1)$ is on the curve.

   (b) Through linearization approximate the y-coordinate on the curve when $x = 1.05$.

   (c) Is this an overestimate or an underestimate? Justify your answer.

5. (11 points) Consider the parametric curve defined by the equations

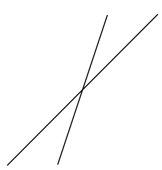
$$x(t) = t^2$$
$$y(t) = t^3 + 2t^2 - t$$

(a) Find the slopes of the tangent lines to the curve at $t = -1$ and at $t = 1$.

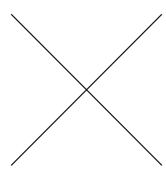
(b) The curve crosses itself at the point where $t = -1$ and $t = 1$. What is this point?

(c) Suppose we zoom in on the curve at the point from part (b). Which of the following pictures most resembles what the curve looks like near this point? Explain using derivatives why you chose your answer.

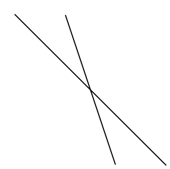
PICTURE A



PICTURE B



PICTURE C

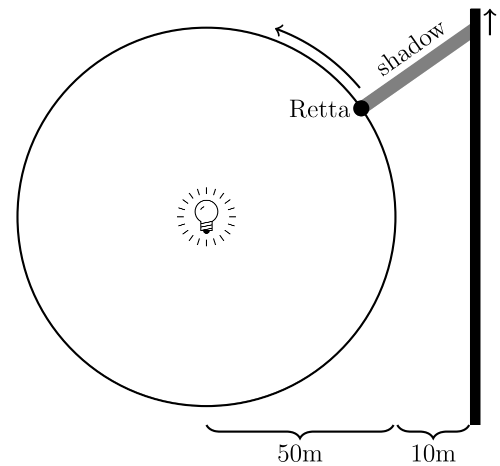


(d) For the picture you chose in part (c), draw arrows on the picture to indicate the direction the curve is traced as t increases. Use the values of dx/dt and dy/dt when $t = -1$ and $t = 1$ to explain your answer.

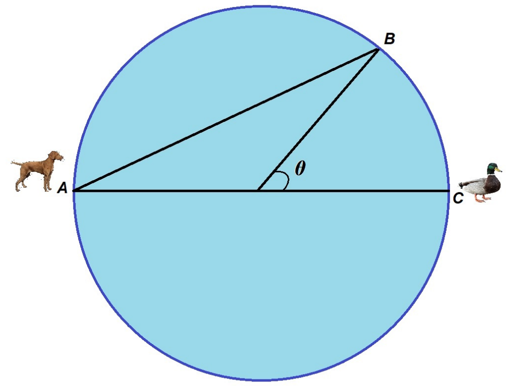
6. (10 points) Retta runs at a speed of 4 meters per second around a circular track of radius 50 meters.

   At the center of the track is a lightbulb, and 10 meters away from the track is a wall.

   When Retta is 35 meters from the wall, how fast is her shadow moving along the wall?



7. (11 points) Copper is standing at the westernmost point of the lake with radius 120 meters when he spots a duck across. To get to the duck, he can swim or run around the lake. He swims at a speed of 6 meters per second and runs at a rate of 12 meters per second. If he swims to point B in a straight line and runs around the lake from B to C, find the angle θ shown which will minimize his time to reach the duck. Make sure you justify using calculus why the answer you found gives the minimum time.
Note: The picture is not to scale.



8. (15 points) Consider the function $f(x) = -\frac{1}{6}x^2 + x - \frac{2}{3}\ln x$.

   (a) What is the domain of the function $f(x)$?

   (b) Find the horizontal and vertical asymptotes of $f(x)$, if any.

   (c) Find the critical numbers of $f(x)$.

   (d) Find the intervals where $f(x)$ is increasing and the intervals where $f(x)$ is decreasing.

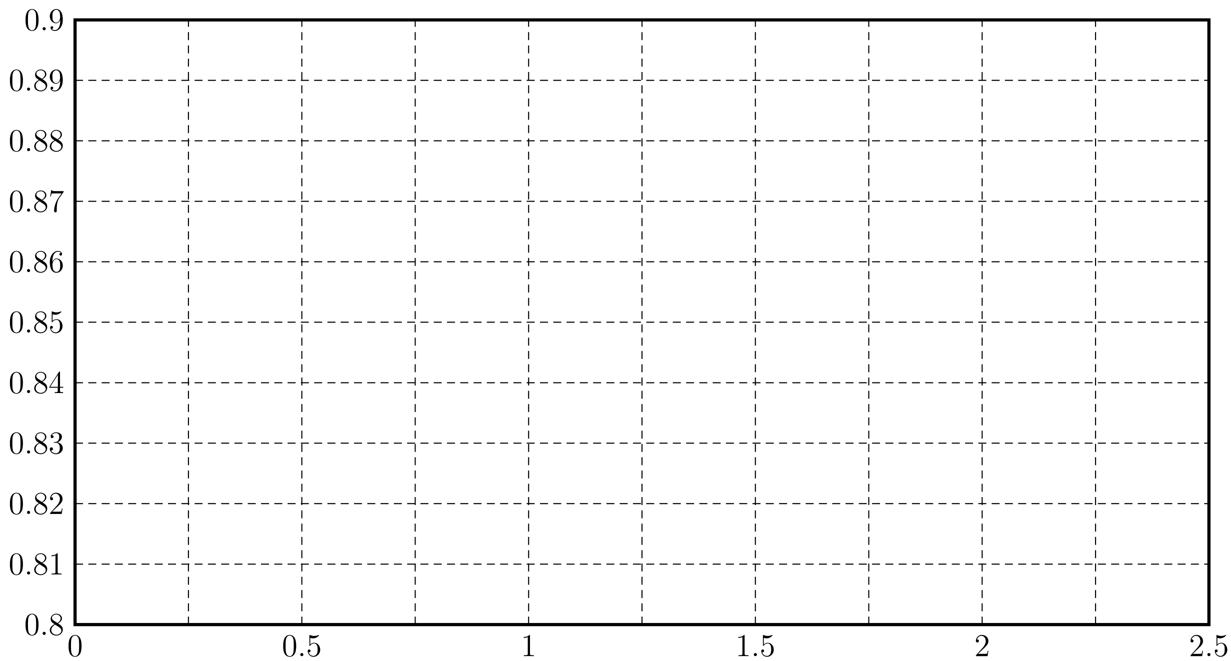
   (e) Find the local maximum and the local minimum values of $f(x)$ (round your answer to 2 decimal places).

Continued on the next page.

Recall that the function is $f(x) = -\frac{1}{6}x^2 + x - \frac{2}{3}\ln x.$

(f) Find the intervals of concavity and the inflection points of $f(x)$ (for inflection points, round your answer to 2 decimal places).

(g) Sketch the graph of $y = f(x)$ on the axis provided below. Be sure to include asymptotes in your picture as well as to mark the coordinates of critical and inflection points.



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