Solutions to Math 120 A Winter 2025 Midterm II

1. With w the width of the rectangle, x the side length of the triangle

$$6w + 3x = 120$$

or $2w + x = 40$. The area is

$$2w^2 + \frac{\sqrt{3}}{4}x^2$$

so we have

$$A(w) = 2w^2 + \frac{\sqrt{3}}{4}(40 - 2w)^2 = (2 + \sqrt{3})w^2 - 40\sqrt{3}w + 400\sqrt{3}$$

or

$$A(x) = 2\left(\frac{40 - x}{2}\right)^2 + \frac{\sqrt{3}}{4}x^2 = \frac{2 + \sqrt{3}}{4}x^2 - 40x + 800.$$

Using the vertex formula

$$w = \frac{20\sqrt{3}}{2 + \sqrt{3}} = 40\sqrt{3} - 60 \quad \text{or} \quad x = \frac{80}{2 + \sqrt{3}} = 160 - 80\sqrt{3}$$

(When you find one, you can find the other using $2w + x = 40$)

2. (a) The population is

$$P(t) = 7500b^t = 7500e^{kt}$$

so

$$3 \cdot 7500 = 7500b^{90} = 7500e^{90k}$$

which gives

$$b = 3^{1/90} \quad \text{or} \quad k = \frac{\ln 3}{90}$$

so the equation is

$$P(t) = 7500 \cdot 3^{\frac{t}{90}} = 7500e^{\frac{\ln 3}{90}t}$$

In three hours,

$$P(180) = 7500 \cdot 3^{\frac{180}{90}} = 7500e^{\frac{\ln 3}{90}180}$$

so

$$P(180) = 7500 \cdot 3^2 = 7500e^{2\ln 3} = 67500$$

(b) In one hour

$$P(60) = 7500 \cdot 3^{\frac{60}{90}} = 7500e^{\frac{\ln 3}{600}180} = 7500 \cdot 3^{2/3}$$

and

$$7500(1 + r) = 7500 \cdot 3^{2/3}$$

so $r = 3^{2/3} - 1 \approx 1.080 = 108\%$.

(c) From

$$2 \cdot 7500 = 7500 \cdot 3^{\frac{t}{90}} = 7500e^{\frac{\ln 3}{90}t}$$

we have

$$2 = 3^{\frac{t}{90}} = e^{\frac{\ln 3}{90}t}$$

taking ln of both sides

$$\ln 2 = \frac{\ln 3}{90}t$$

so

$$t = \frac{90\ln 2}{\ln 3} \approx 57 \text{ minutes.}$$

3. F (even, arms up, no root, shifted, not parabola) $y = (x-3)^6 + 2$

G (even, arms up, no root, parabola) $y = (x-3)^2 + 2$

A (odd, right arm up, one single one double root)$y = (x+3)(x+1)^2$

E (even, has a root, both arms up) $y = x^6 + x^5 - 8x^4 - 12x^3 - 1$

B (odd, right arm up, three roots) $y = (x+2)(x+1)(x-1)$

D (even, both arms down, has roots)$y = -x^4 + 4x^3 - x^2 - 6x + 1$

C (odd, right arm down, only one root) $y = -x^5 - x^4 - 4x^3 - 4x^2 - 4x - 4$

4. (a) From $\ln(x+1) - \ln(x) = 4$ we have

$$\ln\left(\frac{x+1}{x}\right) = 4$$

$$\frac{x+1}{x} = e^4$$

$$x + 1 = e^4 x$$

so

$$x = \frac{1}{e^4 - 1}$$

(b) From $y = \frac{1}{x-4}$ we have $xy - 4y = 1$ so $x = \frac{1+4y}{y}$ which gives

$$g^{-1}(x) = \frac{1+4x}{x}.$$

Therefore,

$$g^{-1}(f(x)) = g^{-1}\left(\sqrt{3x+4}\right) = \frac{1 + 4\sqrt{3x+4}}{\sqrt{3x+4}}.$$

So we must have $3x + 4 > 0$ or $x > -\frac{4}{3}$ so that square root is defined and the denominator is not zero.