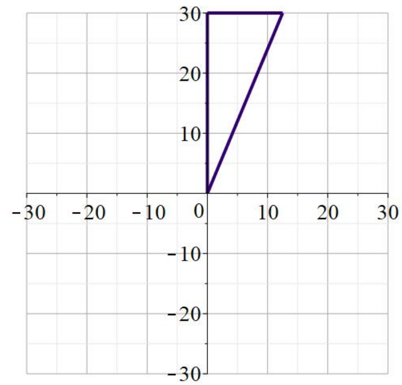
Solutions to Math 120 A Winter 2025 Midterm I

1. (b) He goes North $2.5\times5 = 12.5$ meters and East for $4\times7.5 = 30$ meters and returns for

$$\sqrt{12.5^2 + 30^2} = 32.5 \text{ meters},$$

running for $12.5 + \frac{32.5}{2.6} = 25$ seconds, total of $12.5 + 30 + 32.5 = 75$ meters.



(c)

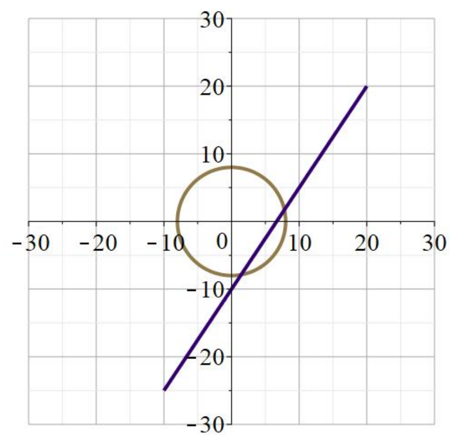
$$f(x) = \begin{cases} 2.5t & \text{if} \quad 0 \le t \le 5 \\ \sqrt{12.5^2 + (4(t-5))^2} & \text{if} \quad 5 \le t \le 12.5 \\ -2.6t + 65 & \text{if} \quad 12.5 \le t \le 25 \end{cases}$$

(d) First when $2.5t = 10$ so at $t = 4$. Since the circle of radius 10 centered at the origin does not intersect his path going East, the next time will be when $-2.6t + 65 = 10$ so when $t = \frac{55}{2.6} \approx 21.15$ seconds.

2. (b) The equation of her path is

$$y + 25 = \frac{20+25}{20+10}(x+10)$$

or $y = 1.5x - 10$. To find the coordinates where she enters and exists the cloud we solve:



$$x^2 + (1.5x - 10)^2 = 64$$

or $3.25x^2 - 30x + 36 = 0$ which gives

$$x = \frac{30 \pm \sqrt{432}}{6.5} \approx 7.813, 1.418.$$

So the points of intersection are $(7.813, 1.720)$ and $(1.418, -7.873)$ with the distance between them

$$\sqrt{(7.813 - 1.418)^2 + (1.720 + 1.418)^2} \approx 11.529 \text{ miles}$$

which would take $\frac{11.529}{130} \approx 0.886$ hours or 5.3 minutes.

(c) The slope of her path is 1.5. She is closest to Seattle when her path intersects with the line $y = -\frac{1}{1.5}x$ so when

$$-\frac{1}{1.5}x = 1.5x - 10$$

at $\left(\frac{15}{3.25}, -\frac{10}{3.25}\right)$ where the distance to the origin (Seattle) would be

$$\sqrt{\left(\frac{1.5}{3.25}\right)^2 + \left(-\frac{10}{3.25}\right)^2} = \frac{\sqrt{325}}{3.25} \approx 5.55 \text{ miles}$$

3.

$$\frac{f(x+h)-f(x)}{h} = \frac{2(x+h)^2-10(x+h)-(2x^2-10x)}{h} = \frac{2(x^2+2xh+h^2)-10x-10h-2x^2+10x}{h}$$

$$= \frac{2x^2+4xh+2h^2-10x-10h-2x^2+10x}{h} = \frac{4xh+2h^2-10h}{h} = 4x+2h-10$$

and at $h=0$ we get $4x-10$.

4. (a) From $\left|\frac{x}{2}+3\right| = 3x+1$ we get two cases:

- $\frac{x}{2}+3 = 3x+1$ so $2 = \frac{5}{2}x$ or $x = \frac{4}{5}$. We check

$$\left|\frac{x}{2}+3\right| = \left|\frac{2}{5}+3\right| = \left|\frac{17}{5}\right| = \frac{17}{5}$$

$$3x+1 = \frac{12}{5}+1 = \frac{17}{5}$$

so this is one solution.

- $\frac{x}{2}+3 = -(3x+1)$ so $\frac{7x}{2} = -4$ or $x = -\frac{8}{7}$. We check

$$\left|\frac{x}{2}+3\right| = \left|-\frac{4}{7}+3\right| = \left|\frac{17}{5}\right| = \frac{17}{5}$$

$$3x+1 = -\frac{24}{7}+1 = -\frac{17}{5}$$

this does not work.

So we only have one solution $x = \frac{4}{5}$.