Math 120 Section   **Midterm I, January 25th**   **Winter 2024**

HONOR STATEMENT

I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.

Name

Signature

Student ID #

- Silence your phone and put it away.

- You have 50 minutes for 4 problems. Check your copy of the exam for completeness.

- You are allowed to use a hand written sheet of paper (8x11 in), back and front.

- Calculator : TI 30 XIIS.

- Justify all your answers and show your work for credit.

- All answers must be exact, no rounding.

- Each problem is worth 10 points.

Do not open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!

Problem 1. A company produces cellphone cases. The production cost follows a **quadratic function model**.

Production cost of 1 box of cellphone cases is $4.50 while the cost of 5 boxes of cellphone cases is $12.50. Because of fixed costs, the production of 0 boxes of cellphone cases is $7.50.

(a) Write the function $f(x)$ of the production cost, where x is the number of boxes of cellphone cases. Put a box around your final answer.

$f(x) = ax^2 + bx + c$

$f(0) = 7.5 = c$

$f(1) = 4.5 = a + b + 7.5 \qquad \rightarrow \quad a = -3 - b$

$f(5) = 12.5 = 25a + 5b + 7.5 \qquad 1 = 5a + b \qquad 1 = -15 - 5b + b$
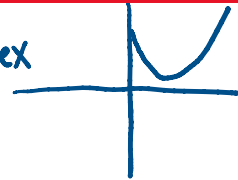
$\Rightarrow \quad b = -4$

$a = 1$

$$\boxed{f(x) = x^2 - 4x + 7.5}$$

(b) How many boxes of cellphone cases should the company produce to minimize production costs? Put a box around your final answer.

minimal cost @ vertex

$x = \dfrac{4}{2} = 2$

Problem 2. A Black Labrador dog is running on a straight line with the same speed to retrieve a ball. The ball is located 90m west and 120m north of the dog's starting point. The dog starts running when $t = 0$ and reaches the ball 15 seconds later. Do not round in this problem.
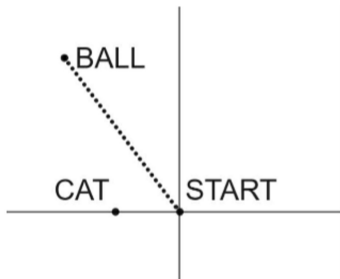
(a) Find the parametric equations of the dog's motion. Put a box around your final answer.

$$x = at + b$$
$$y = ct + d$$

$$@t=0 \left.\begin{array}{l} \\ (0,0) \end{array}\right\} \begin{array}{l} 0 = b \\ 0 = d \end{array}$$

$$@t=15$$
$$(-90, 120)$$

$$-90 = 15a \qquad a = -6$$
$$120 = 15c \qquad b = 8$$

$$\boxed{\begin{array}{l} x = -6t \\ y = 8t \end{array}}$$

(b) When will the dog be due north of a cat who lies 50 m west of the Black Lab's starting point? Put a box around your final answer.

Want $\quad x = -50$:
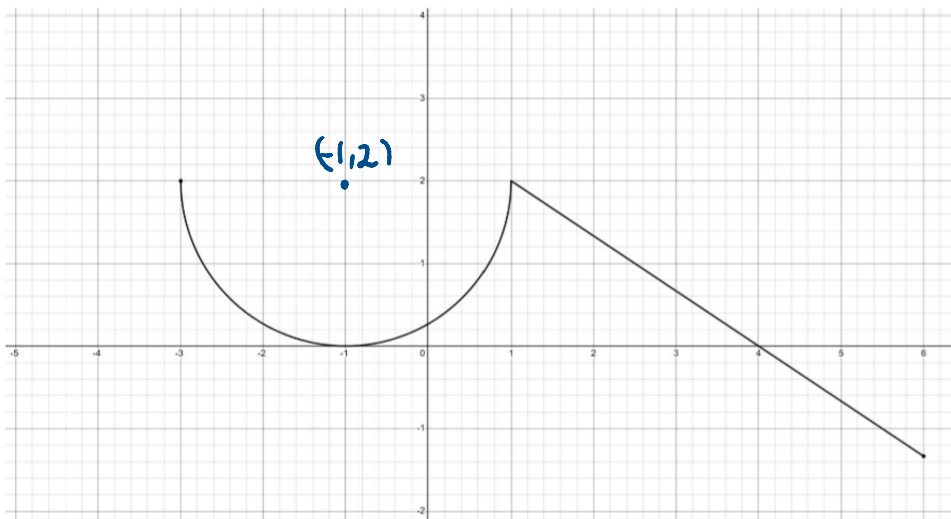
$$-50 = -6t$$

$$\boxed{t = \frac{25}{3} \text{ seconds}}$$

Problem 3. (a) Find the multipart function $f(x)$ whose graph is shown below.

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$$f(x) = \begin{cases} 2 - \sqrt{4-(x+1)^2} & , \quad -3 \le x \le 1 \\ -\frac{2}{3}(x-4) & \quad 1 < x \le 6 \end{cases}$$

2   2

(-1, 2)

(b) What is the range of the function in interval notation?

2

$$\left[-\frac{4}{3}, 2\right]$$

(c) Find the point(s) of intersection of $f(x)$ from (a) with the function $g(x) = -1$ through algebra.

4

$$2 - \sqrt{4-(x+1)^2} = -1$$

$$3 = \sqrt{4-(x+1)^2}$$

$$9 - 4 = -(x+1)^2$$
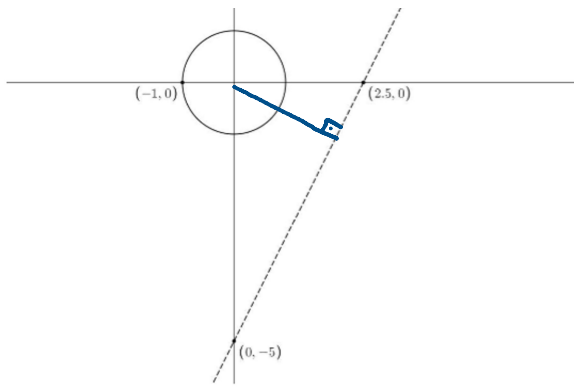
$$-5 = (x+1)^2$$

no solution

$$-\frac{2}{3}(x-4) = -1$$

$$x - 4 = \frac{3}{2}$$

$$x = 5.5$$

$$(5.5, -1)$$

Problem 4. On a circular island of diameter 2km, a Person is stranded. A boat passes by the island on a straight line 5km due south of the island center and a little later 2.5km due east of the island center (see sketch).

(-1, 0)   (2.5, 0)

(a) Where should the Person stand on the island to be closest to the boat when it passes? Find the coordinates, do not round. Put a box around your final answer.

line boat: $y = 2x - 5$

radial line: $y = -\frac{1}{2}x$

6   intersection: $x^2 + y^2 = 1$ $y = \frac{1}{2}x$

$x^2 + \frac{1}{4}x^2 = 1$

$x^2 = \frac{4}{5}$

$x = \frac{2}{\sqrt{5}}$ $x = \frac{2}{\sqrt{5}}$

$y = \frac{1}{\sqrt{5}}$ $\boxed{@ \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)}$

(b) Can the Person draw the boat's attention by shouting from the center of the island if the sound is carried 2.2km?

4   intersection: $-\frac{1}{2}x = 2x - 5$ distance $\sqrt{(2-0)^2 + (-1-0)^2} = \sqrt{5} > 2.2$

$5 = \frac{5}{2}x$ $\boxed{\text{No}}$

$2 = x$

$y = -1$