Midterm 2 Key

Tuesday, February 20, 2024 10:56 AM

Math 120 Section **Midterm II, February 22nd** **Winter 2024**
Instructor: Natalie Naehrig **Section A**

HONOR STATEMENT
I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.

Name

Signature

Student ID #

- Silence your phone and put it away.
- You have 50 minutes for 4 problems. Check your copy of the exam for completeness.
- You are allowed to use a hand written sheet of paper (8x11 in), back and front.
- Calculator : TI 30 XIIS.
- Justify all your answers and show your work for credit.
- All answers must be exact, no rounding.
- Each problem is worth 10 points.

Do not open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!

Problem 1. Consider the function $f(x) = \frac{1}{2}(x+2)^2 - 1$ with domain $-6 \le x \le -2$. The graph of f is shown on the next page.

(a) Compute the range of $f(x)$.

$f(-6) = \frac{1}{2}(-4)^2 - 1 = 8 - 1 = 7$ $\quad f(-2) = \frac{1}{2}\cdot 0^2 - 1 = -1$

range: $-1 \le y \le 7$

(b) Mark the correct circles: The function $f(x)$ has an inverse because it is ● one-to-one ○ onto so it passes the ● horizontal ○ vertical line test.

(c) What are domain and range of $f^{-1}(x)$?

domain: $-1 \le x \le 7$

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domain: $-1 \le x \le 7$

range: $-6 \le y \le -2$

(d) Find the inverse function $f^{-1}(x)$ of $f(x)$.

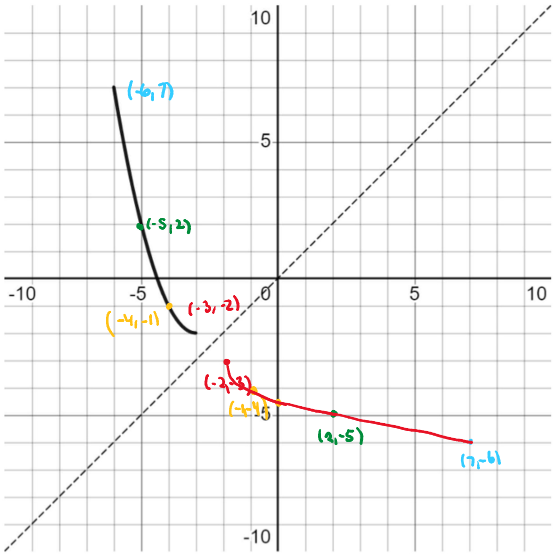
$x = \frac{1}{2}(y+2)^2 - 1 \rightarrow x+1 = \frac{1}{2}(y+2)^2 \rightarrow \pm\sqrt{2(x+1)} - 2 = y$

The range of f^{-1} is $-6 \le y \le -2$, in particular ≤ -2

$\Rightarrow$ negative root:

$f^{-1}(x) = -2 - \sqrt{2(x+1)}$

(e) Using the diagonal $y = x$, sketch the inverse function of $f(x)$ in the given coordinate system. At least 3 points of the graph should be precise.



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Problem 2. The population growth of City A and City B follow an exponential model. At the beginning ($t = 0$, t in years), City A had a population of 24,000 people with a doubling time of 50 years. City B has an initial population of of 50,000 people and grows by 5% in 5 years.

(a) Set up the exponential functions that model City A's and City B's population. The variable t should be in years. Round to 6 decimal places.

City A: $\sqrt[50]{2} = b \approx 1.013959$, $A = 24{,}000$ $\quad f_A(t) = 24{,}000 \cdot 1.013959^t$

City B: $\sqrt[5]{1.05} = b \approx 1.009806$, $A = 50{,}000$ $\quad f_B(t) = 50{,}000 \cdot 1.009806^t$

(b) When will the population of City A be equal to that of City B? Round to the nearest year.

$$f_A(t) = f_B(t) \rightarrow 24{,}000 \cdot 1.013959^t = 50{,}000 \cdot 1.009806^t$$

$$\rightarrow t = \frac{\ln \frac{50}{24}}{\ln \frac{1.013959}{1.009806}} \approx 179 \text{ yrs}$$

(c) What is the doubling time of City B's population? Round to the nearest year.

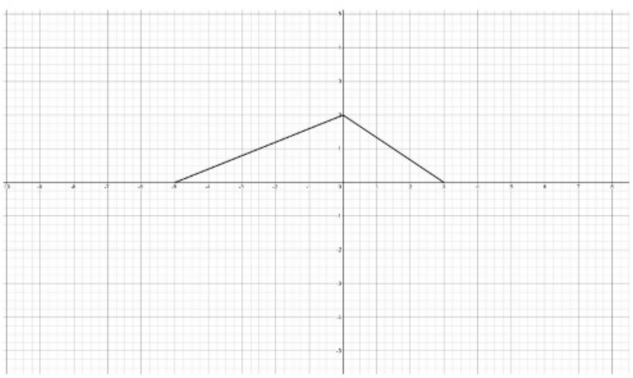
$$T = \frac{\ln 2}{\ln b} \sim 71 \text{ yrs}$$

(d) Assuming that City A's population growth followed the same model before observation started, when was City A's population at 17,000 people? Round to the nearest year

$$17{,}000 = 24{,}000 \cdot 1.013959^t$$

$$t = \frac{\ln \frac{17}{24}}{\ln 1.013959} \sim -25 \text{ yrs}$$

Problem 3. Consider a function $f(x)$ whose graph is shown below.



(a) What are the domain **and** range of $f(x)$?

domain: $-6 \le x \le 3$ range $0 \le y \le 2$

(b) Identify the transformations f has to undergo to obtain $g(x) = -2f(2x-1)$. To do so, mark all circles that apply. By writing numbers 1, 2, 3, ..., in front of the relevant transformations, indicate the order by which they need to be applied. Further, fill any other blanks, if applicable. If a transformation does not apply, leave the circles blank. EDIT

1. ● Horizontal translation by ....1.... units to the ○ left ● right.

○ Vertical translation by .... units to the ○ up ○ down.

2. ● Horizontal dilation by factor ...$\frac{1}{2}$........... (● compression ○ expansion).

3. ~~4.~~ ● Vertical dilation by factor ....2.... (○ compression ● expansion).

4. ~~3.~~ ● Reflection about the x-axis.

also ○ Reflection about the y-axis.

(c) Sketch the graph of $g(x)$ in the given coordinate system on the next page. The original graph as well as a blank coordinate system have been provided. Use the first coordinate system to perform one transformation after the other. Use the blank coordinate system to present your final graph of $g(x)$.

(d) Find the domain of $g(x)$ through algebra and confirm with your graph.

$-5 \le 2x-1 \le 3$

$-4 \le 2x \le 4$

$$-5 \le 2x-1 \le 3$$
$$-4 \le 2x \le 4$$
$$-2 \le x \le 2$$

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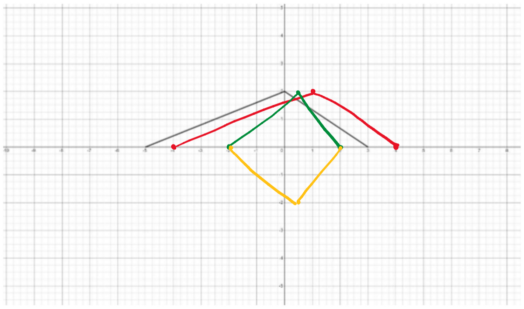

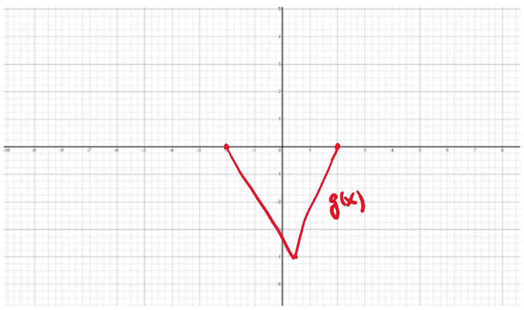

Problem 4. Consider the functions $f(x) = \ln(x-1)$ and $g(x) = \sqrt{2x-4}$.

(a) What are the domains of $f(x)$ and $g(x)$.

$g(x)$: $2x-4 \geq 0$ $\quad 2x \geq 4$ $\quad \boxed{x \geq 2}$

$f(x)$: $x-1 > 0$ $\quad \boxed{x > 1}$

(b) Find $f(g(x))$ and $g(f(x))$.

$f(g(x)) = \ln(\sqrt{2x-4} - 1)$

$g(f(x)) = \sqrt{2 \cdot \ln(x-1) - 4}$

(c) Find the domains of $f(g(x))$ and $g(f(x))$.

$f(g(x))$: $\quad \sqrt{2x-4} > 1 \;\&\; x \geq 2 \longrightarrow 2x-4 > 1 \;\&\; x \geq 2$

$\longrightarrow x > 2.5 \;\&\; x \geq 2 \xRightarrow{\text{strictest}} \boxed{x > 2.5}$

$g(f(x))$ $\quad \ln(x-1) \geq 2 \;\&\; x > 1 \longrightarrow x-1 \geq e^2 \;\&\; x > 1$

$\longrightarrow x \geq e^2+1 \;\&\; x > 1 \xRightarrow{\text{strictest}} \boxed{x \geq e^2+1}$