Final Exam Key

Saturday, March 9, 2024 2:54 PM

Math 120 **Final Exam 03/09** **Winter 2024**
Instructor: Natalie Naehrig **Section A**

HONOR STATEMENT
I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.

Name

Signature

Student ID #

- Silence your phone and put it away.
- You have 170 minutes for 6 problems. Check your copy of the exam for completeness.
- You are allowed to use a hand written sheet of paper (8x11 in), back and front.
- Calculator : TI 30 XIIS.
- Justify all your answers and show your work for credit.
- Unless otherwise instructed, do not round.
- Each problem is worth 10 points.
- The last page can be used for scratch paper work and will not be graded unless otherwise indicated.
- If applicable, Put a box around your final answers.

Do not open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!

Problem 1. Consider the function $f(x) = \frac{1}{4}(x-2)^2 + 1$ with domain $4 \le x \le 9$. The graph of f is shown on the next page.

(a) Compute the range of $f(x)$.

2

$f(4) = \frac{1}{4} \cdot 2^2 + 1 = 2$

$f(9) = \frac{1}{4}(7)^2 + 1 = 13.25$

$\boxed{2 \le y \le 13.25}$

1

(b) Mark the correct circles: The function $f(x)$ has an inverse because it is ● one-to-one ○ onto so it passes the ● horizontal ○ vertical line test.

(c) What are domain and range of $f^{-1}(x)$?

1

domain: $2 \le x \le 13.25$

range: $4 \le y \le 9$

(d) Find the inverse function $f^{-1}(x)$ of $f(x)$.

$x = \frac{1}{4}(y-2)^2 + 1 \rightarrow x - 1 = \frac{1}{4}(y-2)^2$

3

$4(x-1) = (y-2)^2 \rightarrow 2 \pm 2\sqrt{x-1} = y$

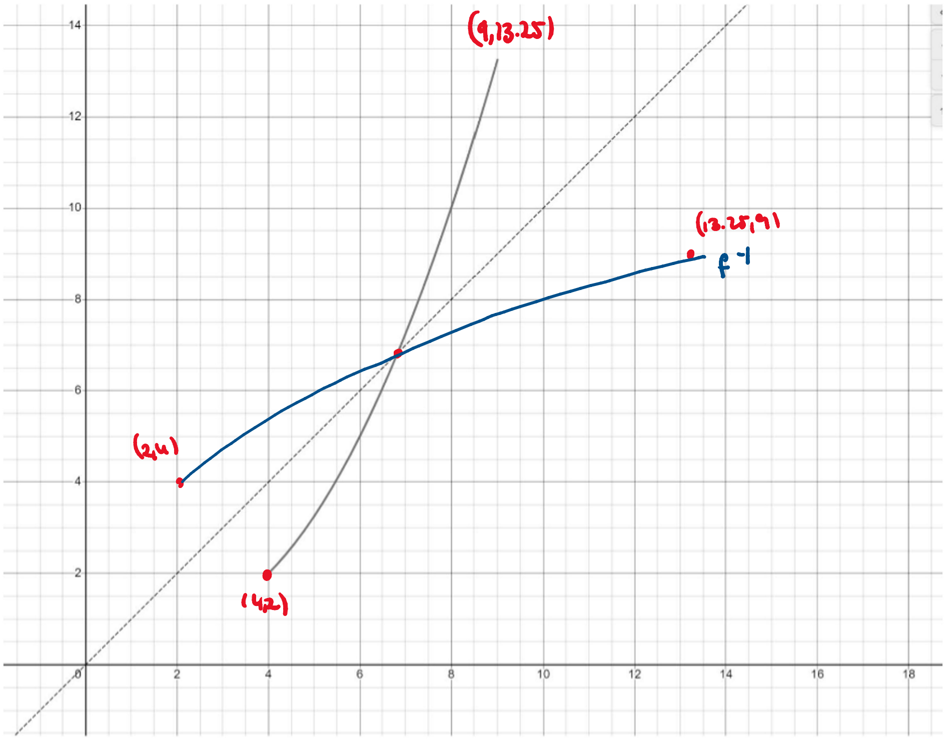
range ≥ 4 so pos root

$\boxed{y = 2 + 2\sqrt{x-1}}$

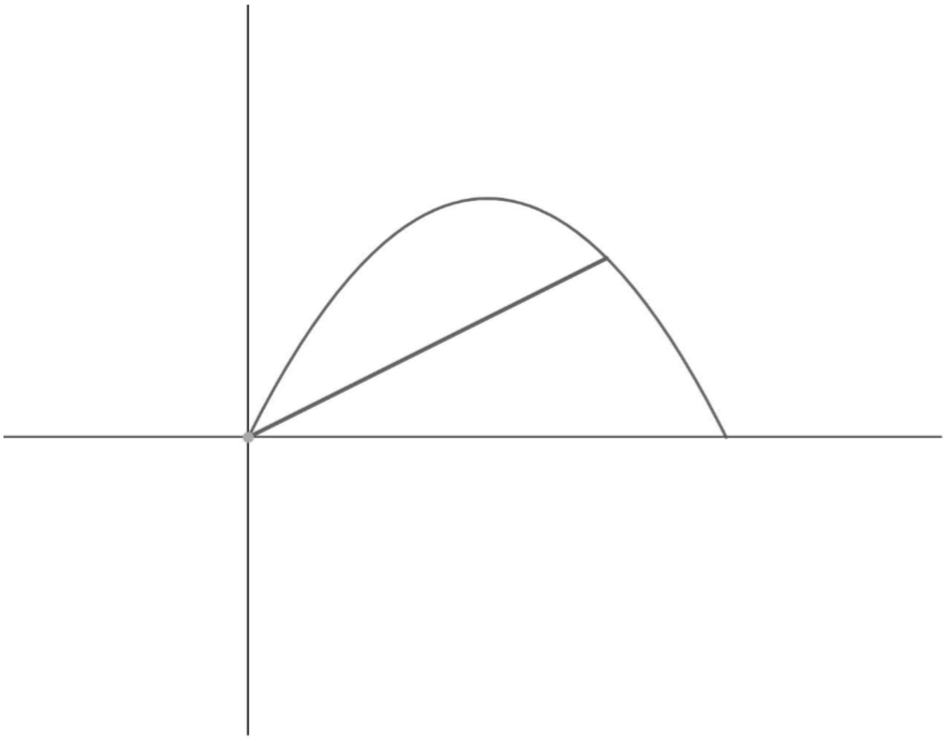
(e) Using the diagonal $y = x$, sketch the inverse function of $f(x)$ in the given coordinate system. At least 3 points of the graph should be precise **and labeled**.

system. At least 3 points of the graph should be precise **and labeled**.

3



Problem 2. Bigfoot has built himself a hiking path in the shape of a parabola. His starting point is the origin. If he hikes 2km East he will be $\frac{10}{3}$km North of the starting point. If he hikes 3km East from the origin, he will be 4.5km North.



(a) Find the quadratic function that models the shape of the path.

$y = ax^2 + bx + c$ $\quad (0,0) \in f \Rightarrow c = 0, (2, \frac{10}{3}) \in f \quad (3, 4.5) \in f$

4

$\frac{10}{3} = 4a + 2b$ $\quad b = \frac{5}{3} - 2a$ $\quad b = \frac{5}{3} + \frac{2}{6} = 2$

$4.5 = 9a + 3b$ $\quad 4.5 = 9a + 3(\frac{5}{3} - 2a)$ $\quad -\frac{1}{2} = 3a \rightarrow a = -\frac{1}{6}$

$\boxed{y = -\frac{1}{6}x^2 + 2x}$

(b) What are the coordinates of the path's most northern point?

vertex @ $x = \frac{2}{2/6} = 6$

2

$y = -\frac{1}{6} \cdot 36 + 12 = -6 + 12 = 6$

$\boxed{(6,6)}$

(c) Bigfoot becomes lazy and builds a shortcut in form of a straight line. The line points 1 meter North for every 2 meters East. Where will Bigfoot join the original path again when he uses the shortcut? 1km = 1000m.

4

$\frac{1m}{2m} = \frac{1km}{2km}$ so slope $\frac{1}{2}$

$y = \frac{1}{2}x$ intersect w/ $y = -\frac{1}{6}x^2 + 2x$

$$\frac{1}{2}x = -\frac{1}{6}x^2 + 2x$$

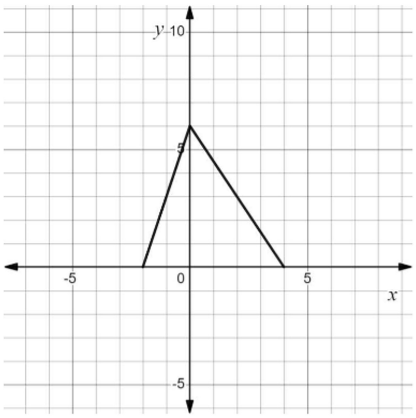
$$\frac{1}{6}x^2 - \frac{3}{2}x = 0$$

$$x\left(\frac{1}{6}x - \frac{3}{2}\right) = 0$$

$x = 0$ OR $x = 9$ $\rightarrow y = 4.5$

$\boxed{@ \ (9, 4.5)}$

Problem 3. Consider a function $f(x)$ whose graph is shown below.



1 (a) What are the domain **and** range of $f(x)$?

domain: $-2 \le x \le 4$ range $0 \le y \le 6$

(b) Identify the transformations f undergoes to obtain $g(x) = -\frac{1}{3}f(2x) + 1$. To do so, mark all circles that apply, leave those blank that do not apply. Write numbers 1, 2, 3, ... in front of the relevant transformations to indicate the order they were applied.

- ○ Horizontal translation by .... units to the ○ left ○ right.
- 4 ● Vertical translation by ....1......... units ● up ○ down.
- 1 ● Horizontal dilation by factor .....½........... (● compression ○ expansion).
- 3 ● Vertical dilation by factor ....⅓........ (● compression ○ expansion).
- 2 ● Reflection about the x-axis.
- ○ Reflection about the y-axis.

5 or 4

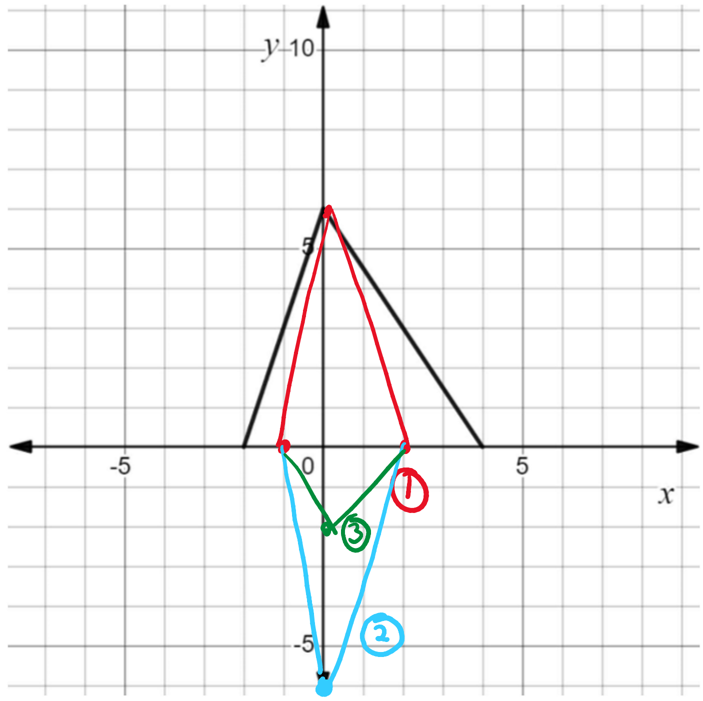
(c) Find the domain of $g(x)$ through algebra.

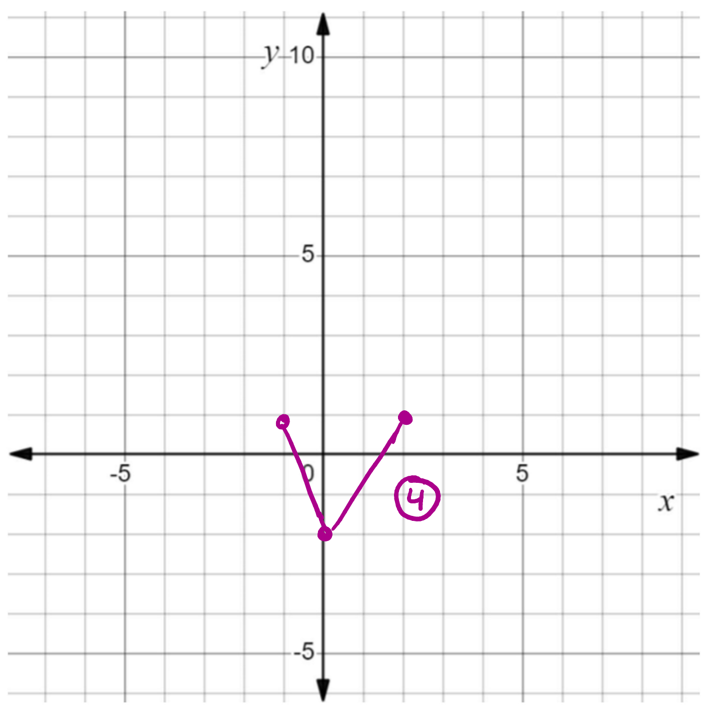
2

$-2 \le 2x \le 4 \rightarrow -1 \le x \le 2$

(d) Sketch the graph of $g(x)$ in the given coordinate system on the next page. The original graph as well as a blank coordinate system have been provided. Use the first coordinate system to perform one transformation after the other and the blank coordinate system to present the final graph of $g(x)$.

2 of 3





Problem 4. A gardener is worried about her cherry trees which are infested with aphids. It seems that the population grows exponentially. In this problem, round each number that represents the aphids to the nearest integer in the final result, but work with 6 decimal places to get there.

(a) She finds that the aphid population triples every 5 days. Initially, (on day 0), there were 100 aphids. Find an exponential function $f_1(t)$ that describes the population t days after the initial day.

3

$$\boxed{f_1(t) = 100\sqrt[5]{3}^{\,t} = 100 \cdot 1.245731^t}$$

$$f_1(8) = 579.954838$$

(b) On day 8 she buys ladybugs and releases them into the cherry trees. Ladybugs feast on aphids and the gardener observes that the number of aphids exponentially decreases by 15% every other day. Set up an exponential function $f_2(t)$ for the declining population of aphids. The independent variable t should be with respect to day 0, when the gardener first noticed the aphids so that the domain of $f_2(t)$ is $t \geq 8$.

3

$$\boxed{f_2(t) = 580 \cdot \sqrt{0.85}^{\,t-8} = 580 \cdot 0.921954^{t-8}}$$

(c) Sketch the multi-part function that describes the aphid population over at least 15 days. Use the provided coordinate system; label the axes.

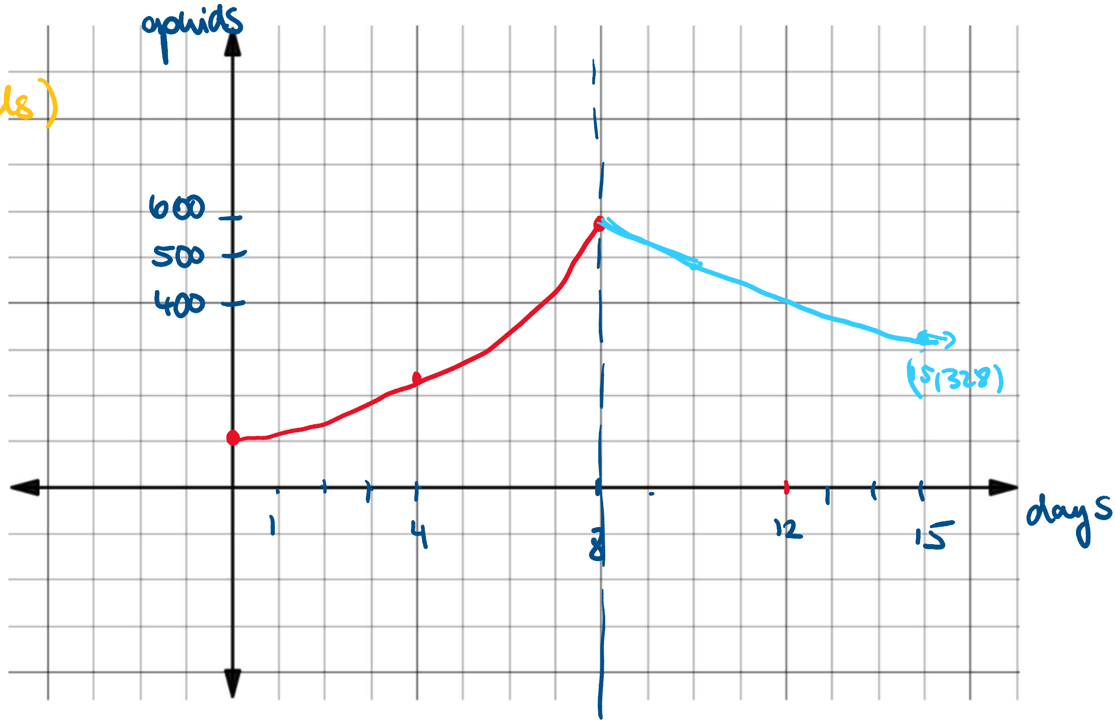
3

aphids

days. Use the provided coordinate system; label the axes.

3

+1 (labels)



Problem 5. Professor Naehrig runs counter-clockwise on a circular track of diameter 90m. She needs 72 seconds to complete a full round.

(a) What is Prof. Naehrig's linear speed?

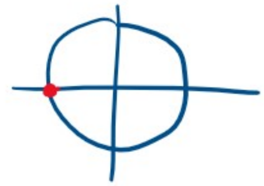
$r = 45\,m$ $\quad V = \omega \cdot 45 = \frac{\pi}{36} \cdot 45 = \boxed{\frac{5}{4}\pi \ \frac{m}{s}}$

2 $\quad \frac{2\pi}{} \quad \frac{\pi}{}$

$r = 45\,m$ $\qquad V = \omega \cdot 45 = \frac{\pi}{36} \cdot 45 = \boxed{\frac{5}{4}\pi \ \frac{m}{s}}$

2 $\qquad \omega = \frac{2\pi}{72} = \frac{\pi}{36}$

(b) Impose a coordinate system with the center as the origin. She starts running at the westernmost point on the track. Write parametric equations for the position of Professor Naehrig after t seconds.



$x_c = y_c = 0$

$\omega = \frac{\pi}{36}$

$r = 45$

4 $\qquad \theta = \pi$

$$\boxed{\begin{aligned} x &= 45\cos\left(\frac{\pi}{36}t + \pi\right) \\ y &= 45\sin\left(\frac{\pi}{36}t + \pi\right) \end{aligned}}$$

(c) When will her x-coordinate be $x = 25\sqrt{2}$ for the first and second time? Round your answer to the nearest second.

$45\cos\left(\frac{\pi}{36}t + \pi\right) = 25\sqrt{2}$

4 $\qquad \cos\left(\frac{\pi}{36}t + \pi\right) = 0.7857$

prin $\qquad \frac{\pi}{36}t + \pi = 0.667 \ +2\pi k \ \rightarrow \ \frac{\pi}{36}t = -2.47 + 2\pi k$

sym $\qquad \frac{\pi}{36}t + \pi = -0.667 + 2\pi k \ \rightarrow \ \frac{\pi}{36}t = -3.81 + 2\pi k$

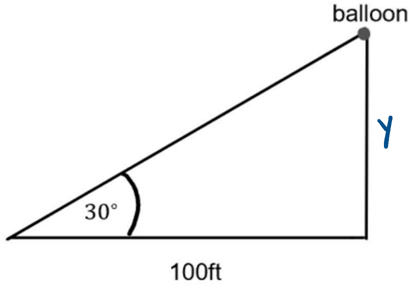
prin $\qquad t = -28.30 + 72k$

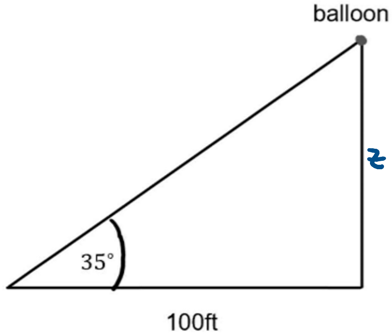
sym $\qquad t = -43.66 + 72k$

k	princ	sym
1	43.7	28.34
2	115.7	100.34

$\boxed{1^{st}: 28\,s \quad 2^{nd}\ 44\,s}$

Problem 6. A person observes a balloon that rises vertically at a constant speed. The horizontal distance of the person and the balloon 100ft. At the first observation, the person measures an angle of observation of 30°, 10 seconds later an angle of 35°. How fast is the balloon rising in feet per second? Round to one decimal place and make sure your calculator is in degree mode.





2+2 $\tan 30° = \frac{y}{100}$ $\tan 35° = \frac{z}{100}$

2 $y = 57.7$ $z = 70.02$

3 + 1 rounding $V = \frac{\text{change in height}}{\text{time needed}} = \frac{12.32}{10} \frac{ft}{s} = \boxed{1.2 \frac{ft}{s}}$