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Monday, January 23, 2023 6:13 PM

Math 120 Section **Midterm I, January 26th** **Winter 2023**

HONOR STATEMENT
I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.

Name

Signature

Student ID #

1.	2.	3.	$\sum$
10	10	10	30

- You have 50 minutes for 3 problems. Check your copy of the exam for completeness.
- You are allowed to use a hand written sheet of paper (8x11 in), back and front.
- Calculator : TI 30 X.
- Justify all your answers and show your work for credit.
- All answers must be exact, no rounding.

Do not open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!

Problem 1. Consider the function $f(x) = |x + 1|$.

(a) Solve the equation $f(x) = 2x + 1$.

(b) Interpret the answer in (a) in the xy-coordinate system using compete sentences. In other words, what would you see at the position of x you found in (a)?

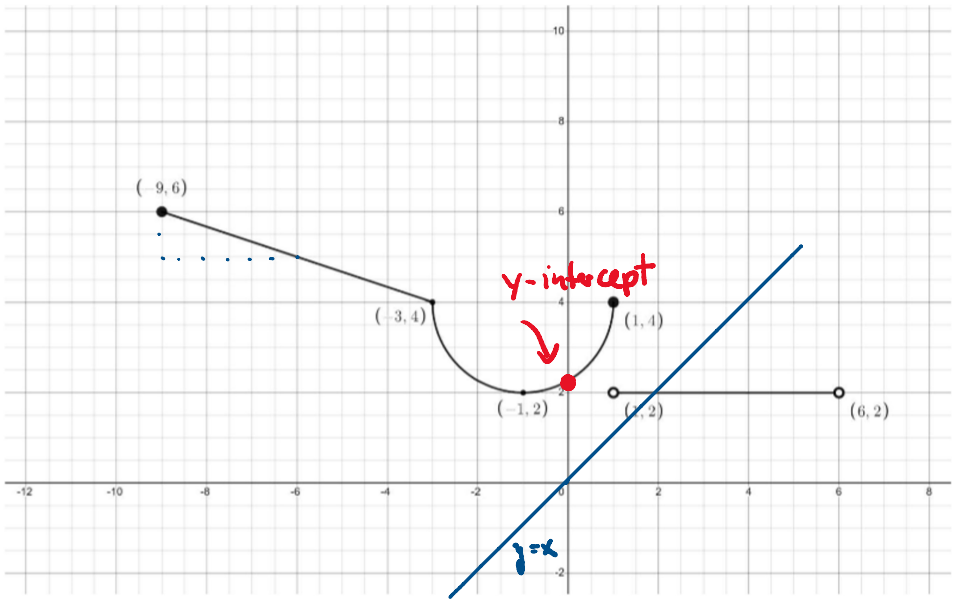
(a) $|x+1| = 2x+1 \Rightarrow \begin{cases} x+1 = 2x+1 & \text{if } x+1 \geq 0 \\ -(x+1) = 2x+1 & \text{if } x+1 < 0 \end{cases} \Rightarrow \begin{cases} x = 0 & \text{if } x \geq -1 \\ 3x = -2 & \text{if } x < -1 \end{cases}$

$\Rightarrow \begin{cases} x = 0 & \text{if } x \geq -1 \checkmark \\ x = -\frac{2}{3} & \text{if } x < -1 \text{ ✗} \end{cases}$ → only one solution: $\boxed{x = 0}$

b) If you intersect the graph of $|x+1|$ with the line $2x+1$ you'll find one point of intersection, namely at $(0, 1)$

answers may vary. At least one strong conclusion.

Problem 2. Consider the following graph of the function $f(x)$.



(a) Find the domain of the function in interval notation.

(b) Write the rule for this multipart function.

(c) In the given coordinate system clearly mark the y-intercept of the graph.

(d) Does the function have x-intercepts? Explain your answer.

(e) Find the range of the function in interval notation.

(f) Does $f(x) = x$ have a solution? You may use the graph to explain your answer.

(g) Is $f(1) = 2$? Explain your answer.

a) $-9 \leq x < 6$

b) $$f(x) = \begin{cases} -\frac{1}{3}(x+3)+4 & \text{if } -9 \leq x < -4 \quad \text{OR } \leq -4 \\ -\sqrt{4-(x+1)^2}+2 & \text{if } -4 \leq x \leq 1 \quad \text{OR } -4 < \\ 2 & \text{if } 1 < x < 6 \end{cases}$$

c) see above

d) f does not have x-intercepts as the graph does not intersect the x-axis.

e) $2 \leq y \leq 6$

f) The line $y=x$ and the graph intersect at exactly one point: $x=2$, so
so $f(x)=2$ has one solution, $x=2$

the closed circle is not with the line, but the open-

so $f(x)=2$ [illegible]

g) $f(1)=4$, as the closed circle is not with the line, but the semi-circle.

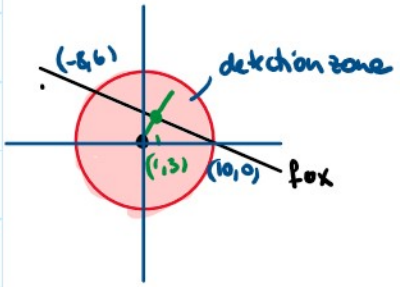
Problem 3. A motion sensor detects all motions around it within 10 feet from its location. Impose a coordinate system whose origin is the location of the detector. A fox trots on a straight line toward the detector, enters the detection zone at $(10, 0)$ and exits it at $(-8, 6)$. Do **not round** in this problem. Do not forget **units in your final answer**.

(a) What distance did the fox cover within the detection zone?

(b) If the speed of the fox is $2\frac{\text{ft}}{\text{s}}$, how long will he be in the detection zone?

(c) What is the closest distance of the fox from the sensor?

(d) Would a rabbit sitting at $(3, -9)$ be detected by the sensor? Explain your answer.



a) $d = \sqrt{(0-6)^2 + (10-(-8))^2} = \sqrt{360} = 6\sqrt{10}$

$\boxed{6\sqrt{10}\text{ ft}}$

b) $v = \frac{d}{t} \Rightarrow t = \frac{d}{v} = \frac{6\sqrt{10}}{2}\frac{\text{ft}}{\text{ft/s}} = \boxed{3\sqrt{10}\,\text{s}}$

c) Find green line equation.

Black line : $y = \frac{0-6}{10+8}(x-10) \rightarrow y = -\frac{1}{3}(x-10)$

Green line: slope = 3 (as perpendicular to black) $\rightarrow y = 3x$

Intersect both lines : $3x = -\frac{1}{3}x + \frac{10}{3} \Leftrightarrow \frac{10}{3}x = \frac{10}{3} \Leftrightarrow x = 1 \rightarrow y = 3$ $(1,3)$

Distance $(1,3)$ from center $(0,0)$: $\sqrt{1+3^2} = \sqrt{10}$

The shortest distance is $\boxed{d = \sqrt{10}\text{ ft}}$

d) Circle equation: $x^2 + y^2 = 100$ Rabbit @ $(3,-9)$

$9 + 81 = 90 < 100 \Rightarrow$ The rabbit is inside the circle

and (with its movement of the mouth and ears) it will be detected.