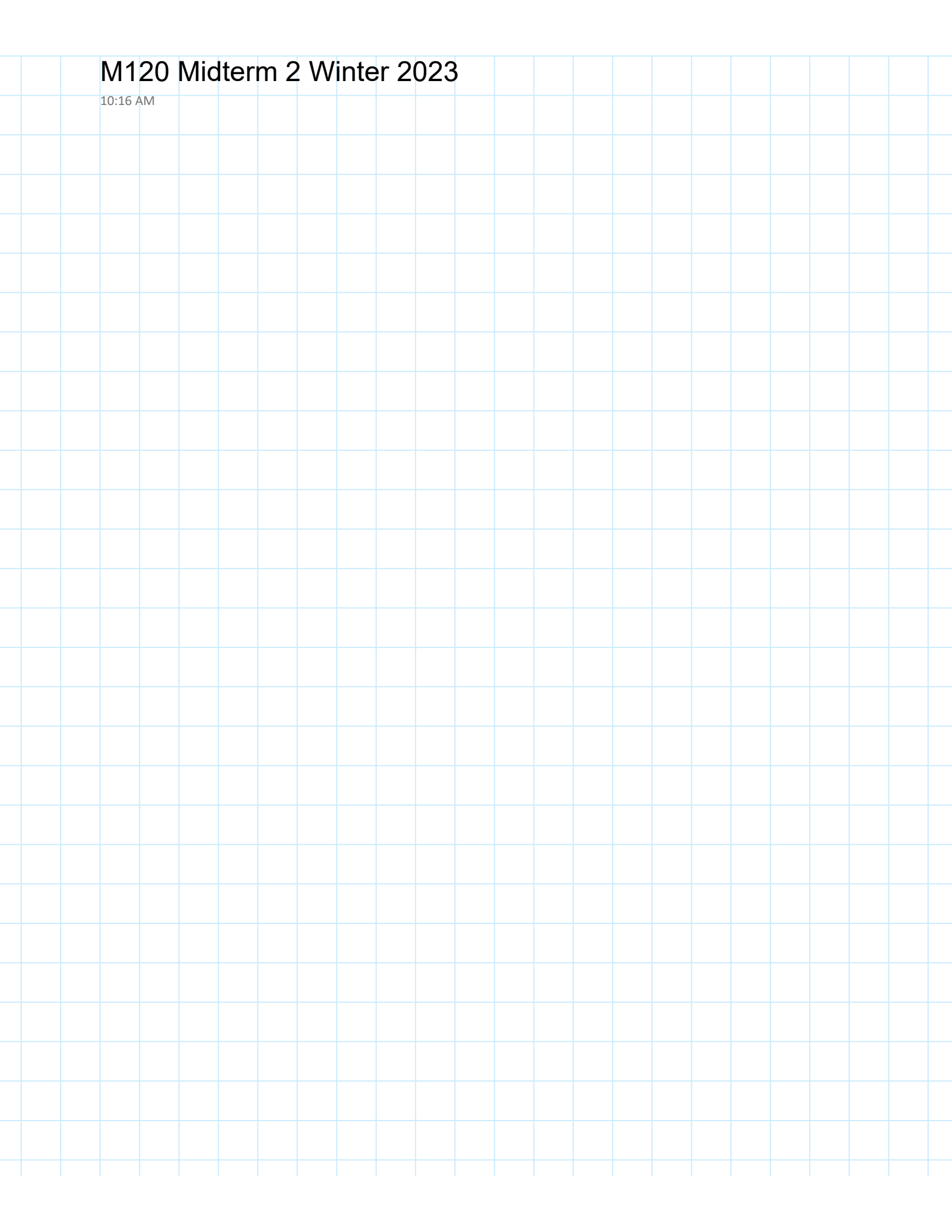# M120 Midterm 2 Winter 2023

10:16 AM

HONOR STATEMENT

I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.

Name

Signature

Student ID #

1.	2.	3.	Σ
10	10	10	30

- You have 50 minutes for 3 problems. Check your copy of the exam for completeness.

- You are allowed to use a hand written sheet of paper (8x11 in), back and front.

- Calculator : TI 30 X.

- Justify all your answers and show your work for credit.

- All answers must be exact, no rounding.

Do not open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!

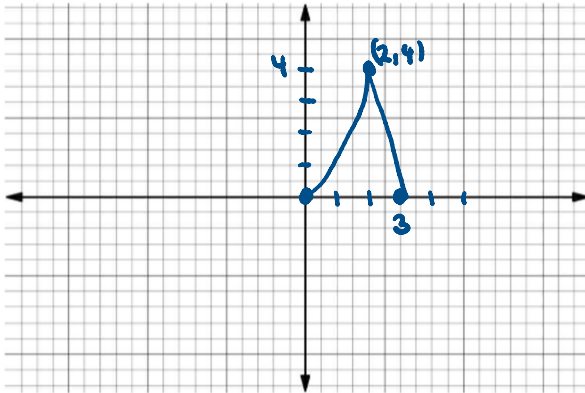**Problem 1.** Consider the function $f(x)$ whose graph is given.
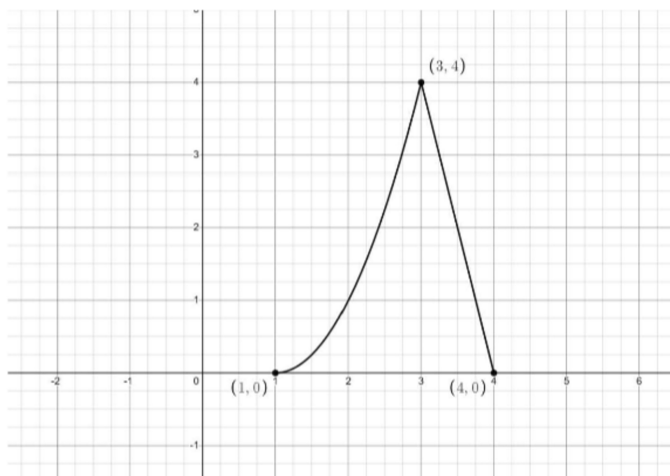
(a) Graph of f

① (a) What is $f(f(2))$? $\quad f(2) = 1 \to f(f(2)) = f(1) = 0$

(r) (b) Is $f(x)$ one-to-one? Explain your answer. **No, because it fails the horizontal line test**

(c) If $g(x) = \frac{1}{4}f\left(-\frac{1}{2}x + 1\right)$, which four transformations has f undergone and in which order? **and by how many units / by which factor ?**

④   I: hor. shift to the left by 1 unit

    II: horiz. dilation by factor 2

    III: reflection about y-axis

    IV: vert dilation by factor $\frac{1}{4}$

(d) What is the domain of $g(x)$? $1 \le x \le 4 \xrightarrow{\text{I}} 0 \le x \le 3 \xrightarrow{\text{II}} 0 \le x \le 6 \xrightarrow{\text{III}} -6 \le x \le 0$ **take out**

(e) What is the range of $g(x)$? $0 \le y \le 4 \xrightarrow{\text{IV}} \boxed{0 \le y \le 1}$

below

(f) In the coordinate systems (b)-(e), sketch the graph of the transformed f after each step you indicated in (c). The final graph (c) should be that of $g(x)$. You must **label all axes, mark units and you must make sure that the three given points** $(1,0), (3,4), (9,0)$ appear in their exact coordinates after the respective (sequence of) transformations.
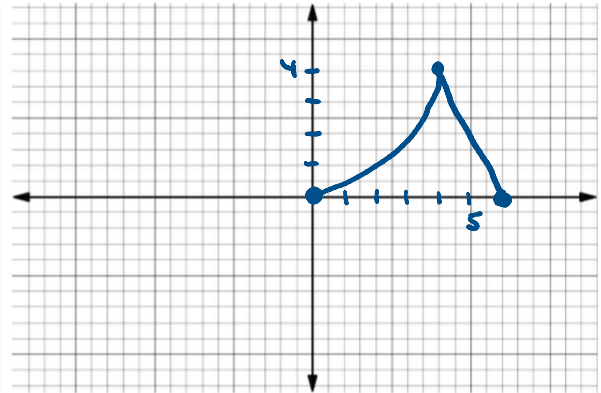
3
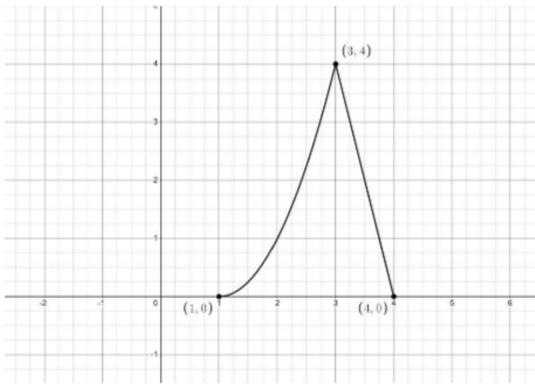
(a) Graph of f - below find $\frac{1}{4}f\left(-\frac{1}{2}x+1\right)$

$(1,0) \xrightarrow{\text{I}} (0,0) \xrightarrow{\text{II}} (0,0) \xrightarrow{\text{III}} (0,0) \xrightarrow{\text{IV}} (0,0)$

$(3,4) \xrightarrow{\text{I}} (2,4) \xrightarrow{\text{II}} (4,4) \xrightarrow{\text{III}} (-4,4) \xrightarrow{\text{IV}} (-4,1)$

$(4,0) \xrightarrow{\text{I}} (3,0) \xrightarrow{\text{II}} (6,0) \xrightarrow{\text{III}} (-6,0) \xrightarrow{\text{IV}} (-6,0)$
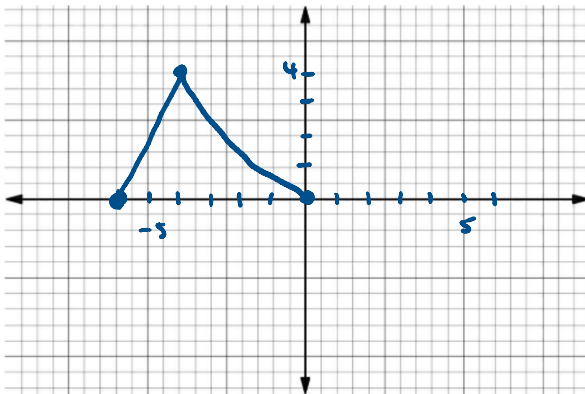

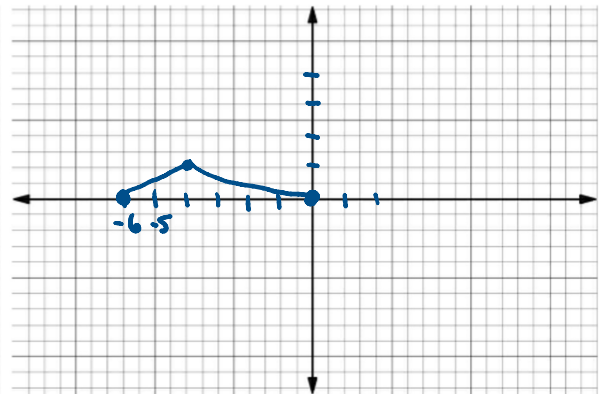(b) Graph after first transformation

(c) Graph after second transformation

(d) Graph after third transformation

(e) Graph after final transformation

4

Problem 2. Town A has 70,000 inhabitants in 2020 and it is expected to grow by 4% every three years. Town B has 60,000 inhabitants in 2020 and grows exponentially so that in 2023 it has 66,000 inhabitants. ~~When~~ will Town B and Town A have the same number of inhabitants?

$y = A \cdot b^x$

In which year

trending

- Town A:

③

$A = 70{,}000$

$b = \sqrt[3]{1.04}$

$f_A(x) = 70{,}000 \cdot \sqrt[3]{1.04}^{\,x} = 70{,}000 \cdot 1.013159^x$

- Town B:

③

$A = 60{,}000$

$\begin{array}{l} 64000 = 60{,}000 \cdot b^3 \\ \dfrac{66}{60} = b^3 \qquad b = \sqrt[3]{\dfrac{11}{10}} \end{array} \Bigg\}$ $f_B(x) = 60{,}000 \cdot 1.03228^x$

-

$f_A(x) = f_B(x)$

$70{,}000 \cdot 1.013159^x = 60{,}000 \cdot 1.03228^x$

③

$\dfrac{7}{6} = \left(\dfrac{1.032280}{1.013159}\right)^x \qquad \Big|\, \ln$

$\ln \dfrac{7}{6} = x \cdot \ln\left(\dfrac{1.03228}{1.013159}\right)$

$8.24 = x$ $\qquad$ (x counted from 01/2020)

①

In 2028 they have equal #s of inhabitants.

6

Problem 3. (a) Given the function $f(x) = \dfrac{1}{1+(x-1)^2}$ on the domain $x \le 1$. Find the rule of the inverse function of $f(x)$.

$x = \dfrac{1}{1+(y-1)^2} \longrightarrow \dfrac{1}{x} = 1+(y-1)^2 \longrightarrow \dfrac{1}{x}-1 = (y-1)^2 \longrightarrow$

$$y = 1 - \sqrt{\tfrac{1}{x}-1} \qquad \text{or} \qquad y = 1 + \sqrt{\tfrac{1}{x}-1}$$

The domain of f is $x \le 1$, so the range of f^{-1} is $y \le 1$

only $\boxed{y = 1 - \sqrt{\tfrac{1}{x}-1}}$ can give this range .

(b) In the following coordinate system you find the graph of a function $g(x)$ (solid line) as well as the line $y = x$ (dashed line). In the coordinate system sketch the inverse function of $g(x)$.

8