Winter 2022 Su

MATH 120 Midterm Exam 1

Tuesday February 1, 50 minutes during your quiz section:
AA: 8:30-9:20am @SMI 405; AB: 9:30-10:20am @SMI 405; AD: 9:30-10:20am @SIG 227
BA: 10:30-11:20am @SMI 405; BB: TuTh 11:30am-12:20pm @SMI 405

Name: | | Quiz section: | |

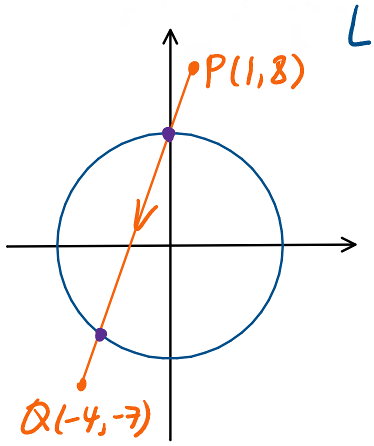
7-digit UW ID: | |

Exam Instruction:

- **You have 50 minutes to complete the four questions.** Distribute your time wisely.
- Show your work to earn full credit. If you can not completely solve a problem, providing reasonable work and steps may still earn you some partial credit.
- Box $\boxed{\text{your final answer}}$ for each problem.
- The last page is a scratch paper, tear it off and do NOT turn it in unless you have written down additional work on it to be graded.
- **Do NOT write within 1 cm of the edge!** Your exam will be scanned for grading. If you run out of space, specify "see scratch paper", then write you additional work on the scratch paper and turn it in together with your exam.
- You can prepare one hand-written double-sided 8.5" × 11" page of notes and bring it to the exam.
- You may use a basic calculator that can NOT graph or solve equation (e.g., TI-30X IIS). All other electronic devices (e.g., cell phone, earbuds) are not allowed, and should be turned off and put away for the duration of the exam.
- You must finish the exam independently. **Giving or receiving any assistance on the exam is considered cheating, which will result in a grade of zero for the exam.** There are multiple versions of the exam, do not discuss the exam questions with other students on the exam day.

1. (15 pts) A motion sensor can detect movement in a circular region with radius 5. Let the center of the circle be the origin of the xy–plane. A racoon walks along a straight path from the point $(1, 8)$ to the point $(-4, -7)$.

(a) Find the point at which the raccoon enters the circular region and the point at which it exits the circular region. The numbers in your answer should be exact, do not use calculator to round to decimal numbers.



L: line through pts $P(1,8)$ & $Q(-4,-7)$

$$\text{slope} = \frac{-7-8}{-4-1} = \frac{-15}{-5} = 3$$

line eqn: $y = 3(x-1)+8 \Rightarrow \underline{y = 3x+5}$

circle: $x^2+y^2=5^2$

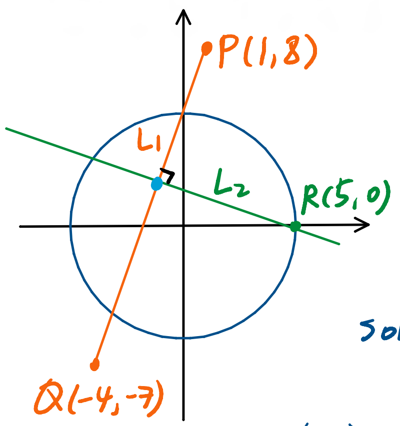
$$x^2+(3x+5)^2=25$$
$$x^2+9x^2+30x+25=25$$
$$10x^2+30x=0$$
$$10x(x+3)=0$$

$\Rightarrow x=0,\ y=3(0)+5=5$

$x=-3,\ y=3(-3)+5=-4$

$\boxed{\begin{array}{l}(0,5)\text{ enter}\\(-3,-4)\text{ exist}\end{array}}$

(b) At which point along the path is the raccoon closest to the point $(5, 0)$? The numbers in your answer should be exact, do not use calculator to round to decimal numbers.



find the intersection pt of L_1 & L_2

L_1: $y=3x+5$, slope: $m_1=3$

$L_2 \perp L_1 \Rightarrow$ slope $m_2 = -\frac{1}{m_1} = -\frac{1}{3}$

L_2: $y=-\frac{1}{3}(x-5)$

solve $L_1 = L_2$

$$3x+5 = -\tfrac{1}{3}(x-5)$$

$(\times 3)$ $9x+15 = -(x-5)$

$10x=-10 \Rightarrow x=-1$

$y=3(-1)+5=2$

$\boxed{(-1,2)}$

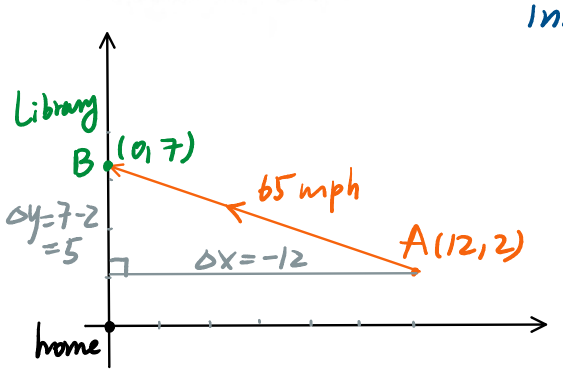
2. (12 pts) Ava is 12 miles due east and 2 miles due north of her home, she starts driving along a straight path towards the library which is 7 miles north of her home at a speed of 65 miles per hour. Let Ava's home be the origin of the xy–plane.

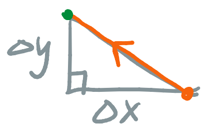
(a) Find parametric equations for the x and y coordinates of Ava t hours after she started driving. The numbers in your answer should be exact, do not use calculator to round to decimal numbers.

initial coord. of Ava: $(x=12, y=2)$

$V = 65$ mph. $\quad x = 12 + V_x t$

$y = 2 + V_y t$





$\Delta x = 0-12 = -12$, $\Delta y = 7-2 = 5$

$$V_x = V\left(\frac{\Delta x}{\sqrt{(\Delta x)^2+(\Delta y)^2}}\right) = 65\left(\frac{-12}{\sqrt{12^2+5^2}}\right)$$

$$= 65\left(\frac{-12}{13}\right) = -60$$

$$V_y = V\left(\frac{\Delta y}{\sqrt{(\Delta x)^2+(\Delta y)^2}}\right) = 65\left(\frac{5}{\sqrt{12^2+5^2}}\right)$$

$$= 65\left(\frac{5}{13}\right) = 25$$

$$\Rightarrow \boxed{\begin{aligned} x &= 12 - 60t \\ y &= 2 + 25t \end{aligned}}$$

(b) Find the distance (in miles) between Ava and her home 10 minutes after she started driving. Round your answer to have 2 decimal places.

dist between Ava: $\begin{cases} x = 12 - 60t \\ y = 2 + 25t \end{cases}$ and home $(x=0, y=0)$

$$\text{dist} = \sqrt{(12-60t)^2 + (2+25t)^2} = \sqrt{\left(12 - \frac{60}{6}\right)^2 + \left(2 + \frac{25}{6}\right)^2}$$

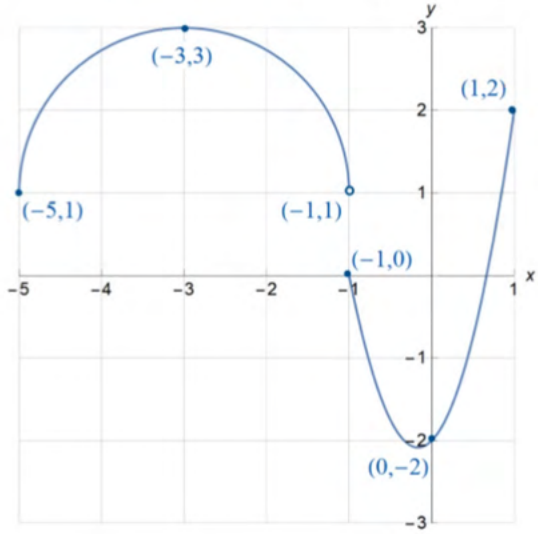
10 min $= \frac{10}{60} = \frac{1}{6}$ hr, plug in $t = \frac{1}{6}$

$$= \sqrt{(12-10)^2 + \left(\frac{37}{6}\right)^2}$$

$$= \sqrt{4 + \frac{1369}{36}}$$

$$\approx \boxed{6.48}$$

3. (12 pts) Here is the graph of a piecewise (multipart) function $f(x)$ and six given points, its left piece is a semicircle as in the picture; its right piece is a quadratic function passing through points $(-1,0)$, $(0,-2)$,$(1,2)$. Find the piecewise (multipart) formula for $f(x)$.



quadratic function

$y = ax^2 + bx + c$

$(-1,0)$: $a(-1)^2 + b(-1) + c = 0$ ①

$(0,-2)$: $a(0)^2 + b(0) + c = -2$ ②

$(1,2)$: $a(1)^2 + b(1) + c = 2$ ③

② $\Rightarrow c = -2$, plug in ① & ③:

① $\Rightarrow a - b - 2 = 0$

③ $\Rightarrow a + b - 2 = 2$

$\begin{cases} a - b = 2 \\ a + b = 4 \end{cases}$

$\Rightarrow 2b = 2 \Rightarrow b = 1$

$a = b + 2 = 3$

$\Rightarrow y = 3x^2 + x - 2$

Circle: center $(x_0 = -3, y_0 = 1)$

radius $= 2$

upper semicircle: $y = y_0 + \sqrt{r^2 - (x - x_0)^2}$

$$\boxed{f(x) = \begin{cases} 1 + \sqrt{4 - (x+3)^2} & x < -1 \\ 3x^2 + x - 2 & x \geq -1 \end{cases}}$$

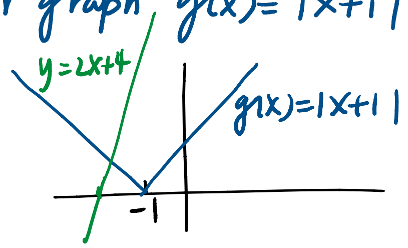
4. (6 pts) Let $g(x) = |x+1|$, solve the equation $g(x) = 2x + 4$.

algebraic approach: $g(x) = |x+1| = \begin{cases} x+1 & \text{if } x+1 \geq 0 \ (x \geq -1) \\ -(x+1) & \text{if } x+1 < 0 \ (x < -1) \end{cases}$

solve $|x+1| = 2x+4$

$\begin{cases} x+1 = 2x+4 \ (\text{if } x \geq 1) \Rightarrow \cancel{x = -3} \ (\text{is } -3 \geq 1 \text{? No!}) \\ -(x+1) = 2x+4 \ (\text{if } x < 1) \Rightarrow \boxed{x = -\frac{5}{3}} \ (\text{is } -\frac{5}{3} < 1 \text{? Yes!}) \end{cases}$

or graph $g(x) = |x+1|$ & $y = 2x+4$



$y = 2x+4$ only intersects the piece $y = -(x+1)$

$\Rightarrow -(x+1) = 2x+4$

$\Rightarrow x = -5/3$