Math 120 **Midterm, January 30th, 2020** **Winter 2020**

HONOR STATEMENT
I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.

Name

Signature

Student ID #

Please circle Section
AA AB AC
BA BB BC

	1.	2.	3.	4.	$\sum$
Possible	15	15	15	15	60
Points					

- Please turn off your cell phone and put it away.
- There are 4 problems on 9 pages. Check your copy of the exam for completeness. Note that front **and** back of the pages are printed on.
- You are allowed to use a hand written sheet of paper (8x11 in), back and front.
- The only calculator allowed is Ti-30x IIS
- When applicable, make a labeled sketch of the situation. It will grant you at least 1 point.
- Justify all your answers and show your work for full credit.
- If you round, then round to the 4th decimal number.

Do not open the test until everyone has a copy and the start of the test is announced.

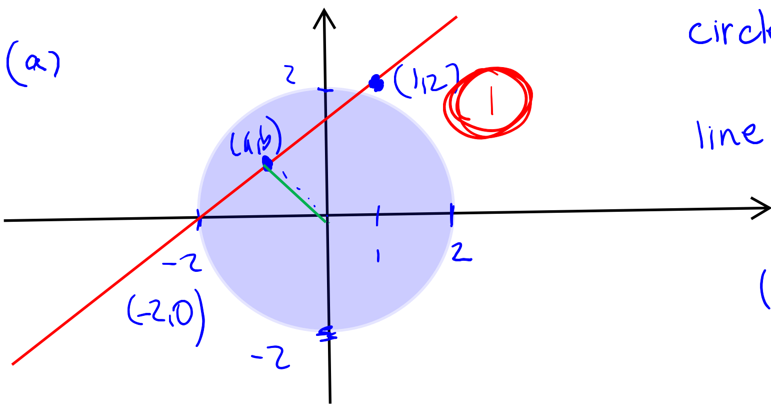
GOOD LUCK!

Problem 1 (~~5~~+~~10~~ points) 7 + 8 *A colorful duck is walking near a lake, which is of the shape of a perfect circle with radius 2km. The duck begins its walk 1km East and 2km North of the center of the lake and it is heading on a straight line to the westernmost point of the lake.*

(a) *Where will the duck enter the lake and start swimming?* Work with exact numbers, no rounding.

(b) *Assume that the duck walks at a speed of 2km/h and swims at a speed of 5km/h. When does the duck reach the closest point to the center of the lake?*

(a)



circle: $\boxed{x^2+y^2=4}$

line: (i) slope : $m = \dfrac{2-0}{1-(-2)} = \dfrac{2}{3}$

(ii) line equation:

$$\boxed{y = \frac{2}{3}(x+2) = \frac{2}{3}x + \frac{4}{3}}$$

intersect line and circle:

$$x^2 + \left(\frac{2}{3}x+\frac{4}{3}\right)^2 = 4 \iff x^2 + \frac{4}{9}x^2 + \frac{16}{9}x + \frac{16}{9} = 4$$

$$\iff \frac{13}{9}x^2 + \frac{16}{9}x = \frac{20}{9} \iff x = -2 \text{ or } \underline{x = \frac{10}{13}}$$

Then $y = \frac{2}{3}\cdot\frac{10}{13} + \frac{4}{13} = \frac{24}{13}$. The duck will enter the lake at $\boxed{\left(\frac{10}{13}, \frac{24}{13}\right)}$

b) Find point on $y = \frac{2}{3}x + \frac{4}{3}$ closest to $(0,0)$:

Perpendicular line through $(0,0)$: $y = -\frac{3}{2}x$

Intersect two lines: $\frac{2}{3}x + \frac{4}{3} = -\frac{3}{2}x \iff x = -\frac{8}{13}$. Then $y = -\frac{3}{2}\cdot\left(-\frac{8}{13}\right) = \frac{12}{13}$ $\left(-\frac{8}{13}, \frac{12}{13}\right)$

Walking from $(1,2)$ to $\left(\frac{10}{13}, \frac{24}{13}\right)$: $d = \sqrt{\left(1-\frac{10}{13}\right)^2 + \left(2-\frac{24}{13}\right)^2} = 0.2774 \longrightarrow \frac{0.2774\,\text{km}}{2\,\text{km/h}} = 0.1387\,h$

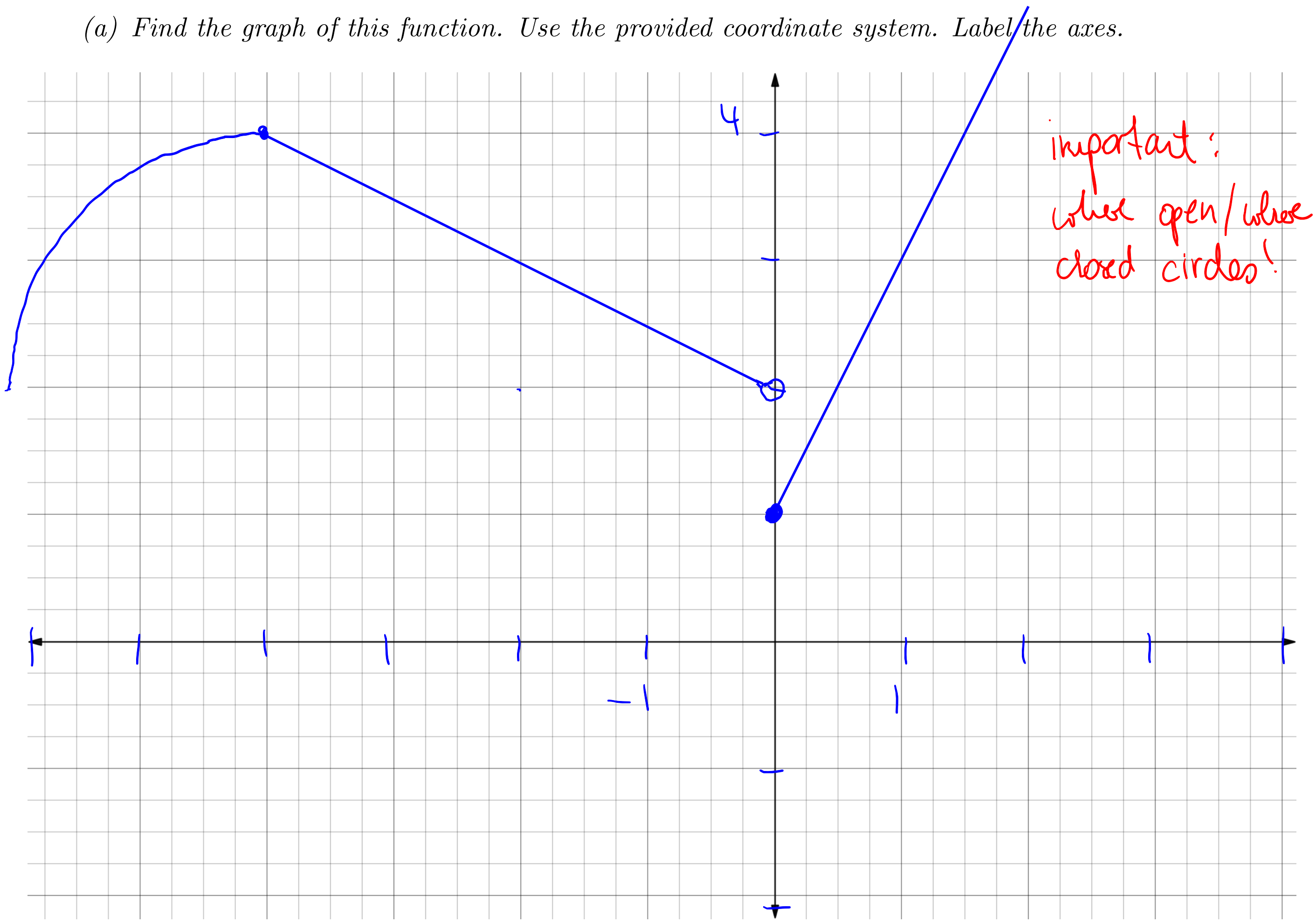
Swimming from $\left(\frac{10}{13}, \frac{24}{13}\right)$ to $\left(\frac{-8}{13}, \frac{12}{13}\right)$: $d = \sqrt{\left(\frac{10}{13}+\frac{8}{13}\right)^2 + \left(\frac{12}{13}-\frac{24}{13}\right)^2} = 1.6641\,\text{km} \longrightarrow \frac{1.6641\,h}{5\,\text{km/h}} = 0.3328\,h.$

Time (total): $0.1387h + 0.3328h = \boxed{0.471\,h}$

Problem 2 (~~7+8~~ 10 points) *Consider the function*

$$f(x) = \begin{cases} \sqrt{4-(x+4)^2}+2 & \text{if} \quad -6 \le x < -4 \\ -\frac{1}{2}x+2 & \text{if} \quad -4 \le x < 0 \\ 2x+1 & \text{if} \quad 0 \le x \le 2 \end{cases}$$

(a) *Find the graph of this function. Use the provided coordinate system. Label the axes.*



Algebraically,

(b) *Find all points where $f(x) = -x$. (More space on the next page)* Exact values, please.

(i) $\sqrt{4-(x+4)^2}+2 = -x \iff \sqrt{4-(x+4)^2} = -2-x \Rightarrow 4-(x+4)^2 = 4+4x+x^2$

$\iff -x^2-8x-16 = 4x+x^2$

$\iff 2x^2+12x=-16 \iff x^2+6x=-8 \iff (x+3)^2 = -8+9 \iff x+3=\pm 1$

$\iff \underbrace{x=-4}_{\text{NOPE}}$ or $\underbrace{x=-2}_{\text{NOPE}}$. ← note: $(-6 \le x < -4!)$

(ii) $-\frac{1}{2}x+2=-x \iff \frac{1}{2}x=-2 \iff x=-4$ YES $\boxed{(-4,4)}$

(iii) $2x+1=-x \Leftrightarrow x=-1$

NOPE

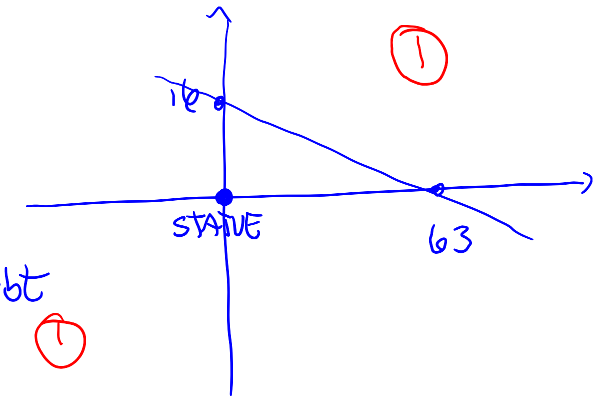
Problem 3 (~~8+7~~ 10 points) *You start running from a point 63 meters due EAST of a statue in a park and run directly toward a point 16 meters due NORTH of the statue. You run at a constant speed of 5 meters per second.*

(a) *Take the statue as the origin of a coordinate system. Find the parametric equations of your location after t seconds of running.*

(b) *A coyote starts running at the same time you do. After t seconds of running, the coyote's location is given by*

$$x = 1 + \frac{2t}{13} \quad y = 55 - \cancel{5t}\ \frac{29}{13}t$$

Find the time t when you and the coyote are closest to each other.



(a) $x = a + bt \quad y = c + dt$ 1

At $t=0$ the location is $(63,0) \rightarrow$

1 $63 = a$ and $0 = c \rightarrow x = 63 + bt$

$y = dt$ 1

We need a second time and corresponding location.

2 Know that you will, at some time, get to $(16,0)$. Distance travelled:

$d = \sqrt{63^2 + 16^2} = 65$. With a speed of 5 m/seconds, this takes 13 seconds to reach. 1

At $t = 13$ the location is $(0,16) \rightarrow$ $0 = 63 + 13b \Rightarrow b = -\frac{63}{13}$ 1

$16 = d \cdot 13 \Rightarrow d = \frac{16}{13}$

2 So $\boxed{x = 63 - \frac{63}{13}t,\ y = \frac{16}{13}t}$

b) Distance between coyote and you at any time: c = coyote y = you

$d = \sqrt{(x_c - x_y)^2 + (y_c - y_y)^2} = \sqrt{\left(1 + \frac{2}{13}t - 63 + \frac{63}{13}t\right)^2 + \left(1 + \frac{29}{13}t - \frac{16}{13}t\right)^2}$

$d = \sqrt{(-62 + 5t)^2 + (1+t)^2} = \sqrt{26t^2 - 618t + 3844}$

d is minimal when parabola under $\sqrt{}$ is minimal, when t is at the vertex: $t = -\frac{b}{2a} = \frac{618}{52} = \boxed{11.8846 \text{ seconds.}}$

Problem 4 (11+4) *Assume the temperature on a sunny day in Seattle (I know...but let's just pretend) can be modeled through a quadratic function. Over the course of the day you measure the temperature three times, 5 hours apart from each other. The first measurement shows 5~~2~~8°F, the second shoes ~~74~~ 75.5°F, and the last shows ~~67~~ 68°F.*

(a) *Determine the quadratic function that fits these three measurement.*

(b) *Assuming your first measurement is at ~~8~~ 9am, when will the temperature reach its maximum?*

(a) $y = ax^2 + bx + c$ where x is time/h and y is temperature. 1

$(0,58)$, $(5,75.5)$, $(10,68)$ are on the graph/elements of the 1

$$\left.\begin{aligned} 58 &= a\cdot 0 + b\cdot 0 + c \\ 75.5 &= a\cdot 5^2 + b\cdot 5 + c \\ 68 &= a\cdot 10^2 + b\cdot 10 + c \end{aligned}\right\} \quad \left.\begin{aligned} c &= 58 \\ 75.5 &= 25a + 5b + 58 \\ 68 &= 100a + 10b + 58 \end{aligned}\right\} \cdot 2 \quad \left.\begin{aligned} &c = 58 \\ &35 = 50a + 10b \\ &10 = 100a + 10b \end{aligned}\right.$$

2

setting up system

$$\left.\begin{aligned} &c = 58 \\ &35 - 50a = 10b \\ &10 = 100a + (35 - 50a) \end{aligned}\right\} \quad \left.\begin{aligned} &c = 58 \\ &35 - 50a = 10b \\ &-25 = 50a \end{aligned}\right\} \quad \left.\begin{aligned} &c = 58 \\ &10b = 35 + 25 \\ &a = -\tfrac{1}{2} \end{aligned}\right\} \quad \left.\begin{aligned} &c = 58 \\ &b = 6 \\ &a = -\tfrac{1}{2} \end{aligned}\right.$$

4

all that

$$\boxed{y = -\frac{1}{2}x^2 + 6x + 58}$$ 1

b) Vertex x-coordinate: $x = -\dfrac{b}{2a} = -\dfrac{6}{2(-\frac{1}{2})} = 6$

4

At 3pm, the temperature will be highest.

Scratch Paper -1

Scratch Paper -2