Math 120 **Midterm II, February 27th, 2020** **Winter 2020**

HONOR STATEMENT
I affirm that my work upholds the highest standards of honesty and academic integrity at the University of Washington, and that I have neither given nor received any unauthorized assistance on this exam.

Name

Signature

Student ID #

Please circle Section
AA AB AC
BA BB BC

	1.	2.	3.	4.	$\sum$
Possible	10	10	15	15	50
Points					

- Please turn off your cell phone and put it away.
- There are 4 problems on 10 pages. Check your copy of the exam for completeness. Note that front **and** back of the pages are printed on.
- You are allowed to use a hand written sheet of paper (8x11 in), back and front.
- The only calculator allowed is Ti-30x IIS
- When applicable, make a labeled sketch of the situation. It will grant you at least 1 point.
- Justify all your answers and show your work for full credit.

Do not open the test until everyone has a copy and the start of the test is announced.

GOOD LUCK!

Problem 1 (10 points) *Consider the quadratic function* $f(x) = 3x^2+9x+2.75$ *with domain* $-\infty \leq x \leq -\frac{3}{2}$.

(a) *What is the range of* f*?*

(b) *Find the inverse function* f^{-1} *of* f.

(a) Put into vertex form: $f(x) = 3(x^2+3x+1.5^2-1.5^2)+2.75$

5 partial credit ok. $= 3((x+1.5)^2 - 2.25)+2.75$

$= 3(x+1.5)^2 - 4$

Vertex is at $(-1.5, -4)$, opening up, so range is

$y \geq -4$

(b) $y = 3(x+1.5)^2 - 4 \rightarrow x = 3(y+1.5)^2 - 4$ 4, partial credit o.k

$\rightarrow \frac{1}{3}(x+4) = (y+1.5)^2 \rightarrow \pm\sqrt{\frac{x+4}{3}} = y+1.5$

$\rightarrow y = -1.5 \pm \sqrt{\frac{x+4}{3}}$. Now the domain of f is $x \leq -\frac{3}{2}$

This means the range of f^{-1} is $x \leq -\frac{3}{2}$

1 → Thus must choose negative root

$\rightarrow \boxed{y = -1.5 - \sqrt{\frac{x+4}{3}} = f^{-1}(x)}$

Problem 2 (10 points) *Consider the function $f(x) = \sqrt{x}$ on the domain $0 \leq x \leq 9$. A graph of f looks like Figure 1 on page 4.*
Through shifting, reflecting and dilation this function was altered to $g(x) = -\frac{1}{2}f(-2x+9)$.

(a) *Find the domain and the range of $g(x)$.* 2

(b) *Describe the manipulations to the graph we did. Add details by using 'compressed/stretched by factor....', 'reflected about the ....-axis', 'shifted right/left/up/down by .... units':*

HORIZONTAL	*shift:*	shift to the left be 9 units
	dilation:	compression by factor 2
	reflection	reflection about y-axis
VERTICAL	*reflection:*	reflection about x- axis
	dilation:	compression by factor 2
	shift	no shift

1 each. no partial credit

(c) *Identify the graph of $g(x)$ among the those on page 5.* 2

a) Domain of f:

$0 \leq x \leq 9$

range of f: $0 \leq y \leq 3$

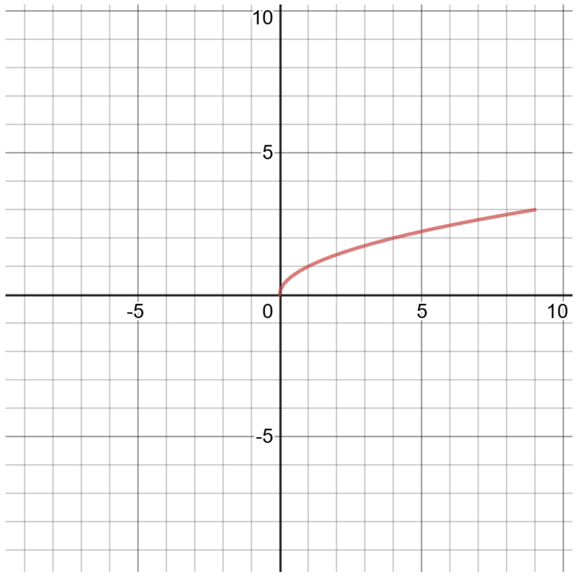
Domain of g: $0 \leq -2x+9 \leq 9$

$-9 \leq -2x \leq 0$

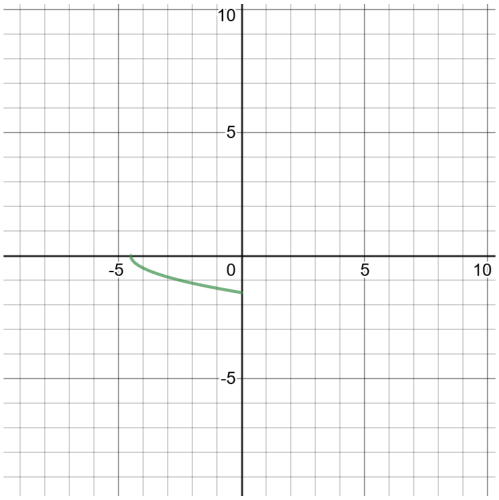
$4.5 = \frac{9}{2} \geq x \geq 0$

domain of g is $0 \leq x \leq 4.5$

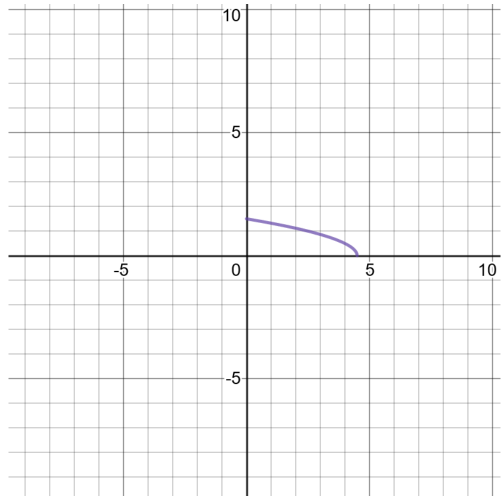
range of g is $-\frac{3}{2} \leq g(x) \leq 0$



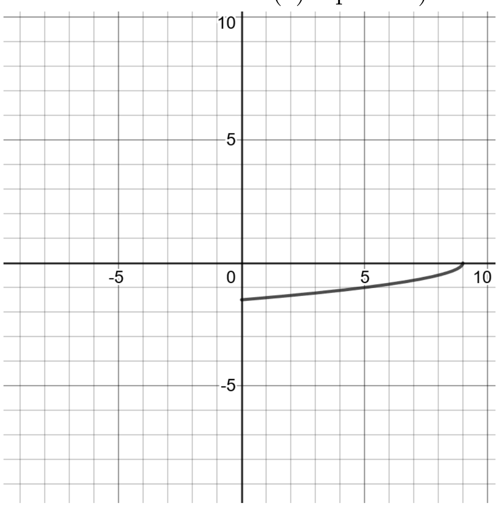
(a) Figure 1



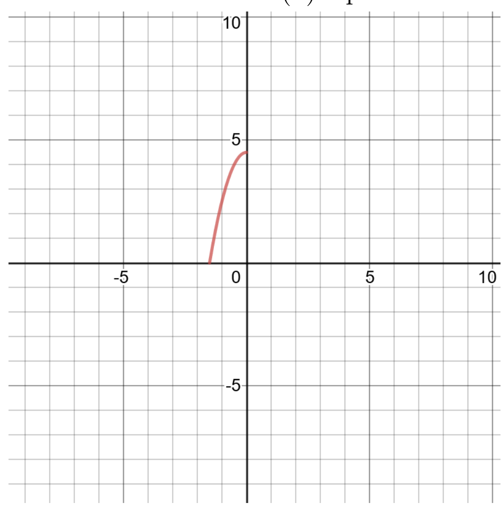
(a) Option a) ☐



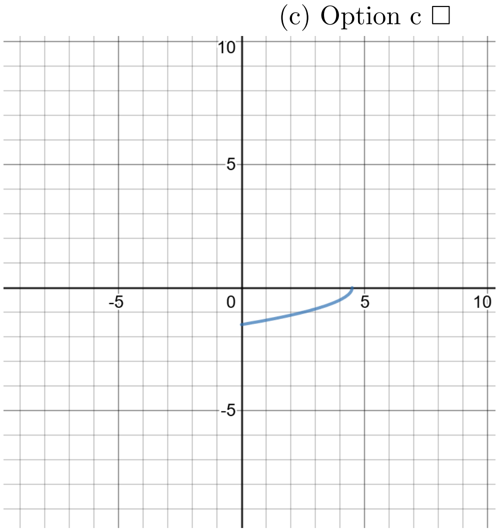
(b) Option b ☐



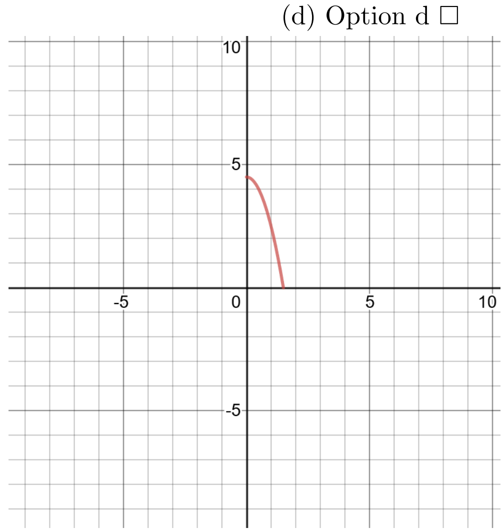
(c) Option c ☐



(d) Option d ☐



(e) Option e ☐



(f) Option f ☐

Problem 3 (15 points) *The population of City A is growing at a constant rate of 2% each year. In the year 2000, this city had 20,000 inhabitants. The population of City B grows exponentially. In 2010, City B had half as many inhabitants as City A at that time. In 2020, City B had 5,000 inhabitants more than City A. In which year did both cities have the same number of inhabitants?*

t = years passed since year 2000

City A: $N_A(t) = 20{,}000 \cdot 1.02^t$ → 1 for constant, 1 for growth rate

City B: $N_B(t) = N_0 \cdot b^t$ 1 template

(*1) $N_B(10) = \frac{1}{2} N_A(10) \rightarrow N_0 \cdot b^{10} = 10{,}000 \cdot 1.02^{10}$ 1

(*2) $N_B(20) = N_A(20) + 5000 \rightarrow N_0 \cdot b^{20} = 20{,}000 \cdot 1.02^{20} + 5000$ 1

Now (*1): $N_0 = \frac{10{,}000 \cdot 1.02^{10}}{b^{10}}$ 1 $\xrightarrow{\text{in (*2)}}$ 1 $\frac{10{,}000 \cdot 1.02^{10}}{b^{10}} \cdot b^{20} = 34718.948$

$\rightarrow b^{10} = \frac{34718.948}{12189.944} = 2.84816$

$\rightarrow b = 1.11$ 1 $\rightarrow N_0 = 4293.109$ 1

$N_B(t) = 4293.109 \cdot 1.11^t$

want: $N_A(t) = N_B(t)$ 1

$4293.109 \cdot 1.11^t = 20000 \cdot 1.02^t$

$\Leftrightarrow \left(\frac{1.11}{1.02}\right)^t = \frac{20000}{4293.109}$ 1

$\Leftrightarrow t \cdot \ln\left(\frac{1.11}{1.02}\right) = \ln \frac{20000}{4293.109}$ 2

$\Leftrightarrow t = 18.2$ 1

$\rightsquigarrow$ $\boxed{\text{in the year 2018}}$ 1

Problem 4 (15 points) *While downloading the new season of their favorite TV show, Family Smile notices a slow-down of the download speed. It turns out that the downloaded percentage is a linear-to-linear rational function in time, where time t is measured in minutes. At first, the download was 0%.*
After 30 seconds, the download was at 18%.
After 4 minutes, the download was at 60%.

(a) *Where will the download be after 20 long minutes?*

(b) *Will it ever finish the download? Explain your answer.*

(a) $f(t) = \frac{at+b}{t+d}$ 1 template

$f(0) = 0 \rightarrow b = 0$ 1

$f(0.5) = 18\% \rightarrow \frac{\frac{1}{2}a}{\frac{1}{2}+d} = 18$ 1 so $a = 18 + 36d$ 1

$f(4) = 60\% \rightarrow \frac{4a}{4+d} = 60$ 1 so $\frac{4(18+36d)}{4+d} = 60$ 1

$\Rightarrow 72 + 144d = 240 + 60d$ 2

$84d = 168$

$d = 2$ 1 $\longrightarrow a = 18 + 36 \cdot 2 = 90$ 1

$\Rightarrow f(t) = \frac{90t}{t+2}$

$f(20) = \frac{1800}{22} = 81.\overline{8}\%$ 1

(b) The asymptote is 90% and the function is increasing and approaching 90%, but never crossing it, implying that 100% will never be reached.

4. Different approaches possible

Extrapaper