Math 120 A - Winter 2016
Midterm Exam Number Two
February 25th, 2016

Name: ______________________________ Student ID no. : ________________

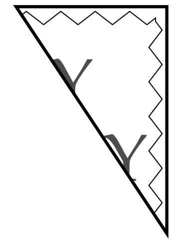
Signature: ____________________________________ Section: ____________

1	15	
2	8	
3	7	
4	15	
5	15	
Total	60	

- This exam consists of FIVE problems on FIVE pages, including this cover sheet.
- Show all work for full credit.
- You may use a scientific calculator during this exam. Graphing calculators are not permitted. Also, other electronic devices are not allowed, and should be turned off and put away for the duration of the exam.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- You may use one hand-written double-sided 8.5″ by 11″ page of notes.
- You have 50 minutes to complete the exam.

∘

1. I'm starting a business selling slices of cake!

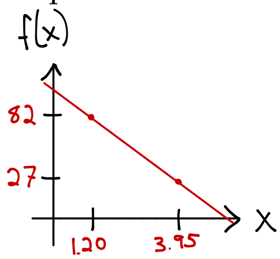


The number of slices of cake I'll sell each day is a **linear** function of its price.

If I charge \$1.20 for a slice, then I will sell 82 slices per day.

If I charge \$3.95 for a slice, then I'll only sell 27 slices per day.

(a) **[6 points]** Give a function $f(x)$ for the *number of slices* I'll sell per day if I charge $\$x$ per slice.



$f(x) = {}^-20(x - 1.20) + 82$

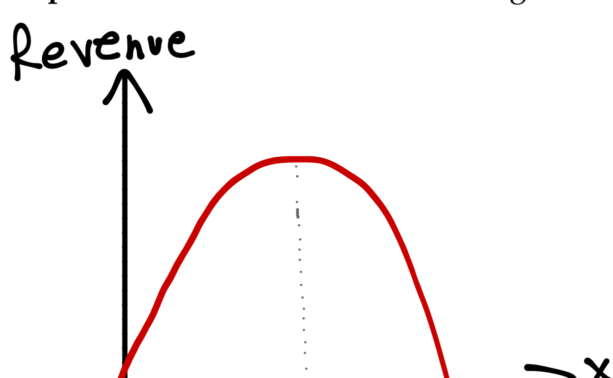
$\boxed{f(x) = {}^-20x + 106}$

(b) **[3 points]** Give a function $g(x)$ for the *total amount of money* I'll make each day if I charge $\$x$ per slice.

$$g(x) = \underbrace{x}_{\text{price per slice}}\underbrace{(-20x + 106)}_{\text{\# slices sold}}$$

$\boxed{g(x) = -20x^2 + 106x}$

(c) **[6 points]** How much should I charge to maximize the amount of money I make?

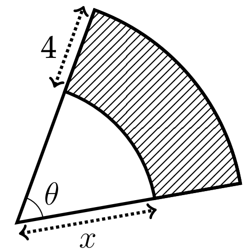


$h = \frac{-b}{2a}$

$= \frac{-106}{2(-20)} = \boxed{\$2.65}$

Due to budget constraints, the following questions are very short.

2. **[8 points]** In the following figure, the area of the shaded region is 9 square units. θ is 0.4 radians. Find x.



Area of shaded region

$= (\text{Area of big sector}) - (\text{Area of small sector})$

$= \frac{1}{2}(0.4)(4+x)^2 - \frac{1}{2}(0.4)x^2 = 9$

$\frac{\frac{1}{2}(0.4)}{\frac{1}{2}(0.4)}\left((4+x)^2 - x^2\right) = \frac{9}{\frac{1}{2}(0.4)}$

$(4+x)^2 - x^2 = 45$

$16 + 8x + x^2 - x^2 = 45 \rightarrow 8x = 29$

$\boxed{x = 3.625}$

3. **[7 points]** Let $f(x) = x^2 + 3^x$. Suppose we take the graph of $y = f(x)$ and do three things:

- First, scale it horizontally by a factor of $\frac{1}{3}$.
- Then, shift it 4 units to the left.
- Then, reflect it vertically over the x-axis.

Write a function $g(x)$ for this new transformed graph.

- $y = x^2 + 3^x$

  $x \to \frac{x}{1/3}$ (or $3x$)

- $y = (3x)^2 + 3^{3x}$

  $x \to x + 4$

- $y = (3(x+4))^2 + 3^{3x}$

  $y \to -1$

- $-y = (3(x+4))^2 + 3^{3(x+4)}$

$y = -\left((3(x+4))^2 + 3^{3(x+4)}\right)$

$\boxed{g(x) = -\left((3(x+4))^2 + 3^{3(x+4)}\right)}$

4. $f(x)$ is a **linear-to-linear rational function** such that the curve $y = f(x)$ passes through the points $(1, -9)$ and $(10, 60)$ and has vertical asymptote $x = 7$.

(a) **[9 points]** Write a formula for $f(x)$.

$$f(x) = \frac{ax+b}{x+d}$$

$$-9 = \frac{a+b}{1+d} \rightarrow -9-9d = a+b \xrightarrow{d=-7} 54 = a+b$$

$$60 = \frac{10a+b}{10+d} \rightarrow 600+60d = 10a+b \xrightarrow{d=-7} -(180 = 10a+b)$$

$$-126 = -9a$$

$$\boxed{a = 14}$$

$$54 = 14 + b$$

$$\boxed{b = 40}$$

$$\boxed{d = -7}$$

$$\boxed{f(x) = \frac{14x+40}{x-7}}$$

(b) **[6 points]** Write a formula for its inverse function, $f^{-1}(x)$.

$$y = \frac{14x+40}{x-7}$$

$$(y-7)x = \frac{14y+40}{y-7}(y-7)$$

$$xy - 7x = 14y + 40$$

$$xy - 14y = 7x + 40$$

$$y(x-14) = 7x + 40$$

$$\boxed{f^{-1}(x) = \frac{7x+40}{x-14}}$$

5. People are moving out of Beattle (mostly because it's filled with bees).

(a) **[4 points]** The population was 500,000 in 2010, and every year it decreases by 3.7%.

Write a function $f(t)$ for the population of Beattle, t years after 2010.

$$f(t) = A_0 b^t \qquad b = 1 - .037 = 0.963$$

$$A_0 = 500{,}000 \qquad \boxed{f(t) = 500000(0.963)^t}$$

(b) **[6 points]** Meanwhile, the number of bees in Beattle is growing exponentially!

In the year 2014, there was one bee for every person in Beattle.

In the year 2025, there will be three bees for every person in Beattle.

Write a function $g(t)$ for the number of bees in Beattle, t years after 2010.

$$g(t) = A_0 b^t$$

$$g(4) = f(4) = 430006$$

$$g(15) = 3f(15) = 852090$$

$$A_0 b^4 = 430006$$

$$A_0 b^{15} = 852090$$

$$b^{11} = 1.981575$$

$$\boxed{b = 1.064145}$$

$$A_0(1.064145)^4 = 430006$$

$$\boxed{A_0 = 335329}$$

$$\boxed{g(t) = 335329(1.064145)^t}$$

(c) **[5 points]** When will there be five million bees in Beattle? Round your answer to the nearest year.

$$\ln\left(335329(1.064145)^t\right) = \ln(5000000)$$

$$\ln(335329) + t\ln(1.064145) = \ln(5000000)$$

$$t = \frac{\ln(5000000) - \ln(335329)}{\ln(1.064145)} = 43.46, \text{ so } \boxed{\text{year } 2053}$$

years after 2010