Math 120 A - Winter 2016
Midterm Exam Number One
January 28th, 2016

Name: _____ Student ID no. : _____

Signature: _____ Section: _____

1	15	
2	15	
3	15	
4	15	
Total	60	

- This exam consists of FOUR problems on FIVE pages, including this cover sheet.
- Show all work for full credit.
- You may use a scientific calculator during this exam. Graphing calculators are not permitted. Also, other electronic devices are not allowed, and should be turned off and put away for the duration of the exam.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- You may use one hand-written double-sided 8.5" by 11" page of notes.
- You have 50 minutes to complete the exam.

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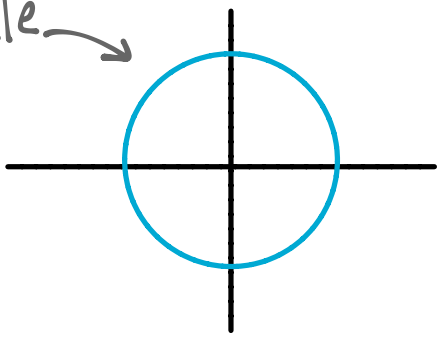
1. Electra stands 40 feet west and 15 feet north of the center of a circular puddle and begins running east at a speed of 10 feet per second.

   After t seconds, the puddle has a radius of $8t$ feet.

   (a) **[12 points]** From when she enters to when she leaves, how much time does Electra spend running through the puddle?

Equation for puddle: $x^2 + y^2 = (8t)^2$

Puddle →



Electra's coordinates after t seconds:

$x = -40 + 10t$

$y = 15$

When is she on the border?

$$(-40+10t)^2 + 15^2 = (8t)^2$$

$$1600 - 800t + 100t^2 + 225 = 64t^2$$

$$36t^2 - 800t + 1825 = 0$$

$$t = \frac{800 \pm \sqrt{800^2 - 4(36)(1825)}}{2(36)} \approx 2.581 \text{ or } 19.641$$

When she enters ↓ 2.581

When she leaves ↓ 19.641

Time in puddle = 19.641 − 2.581

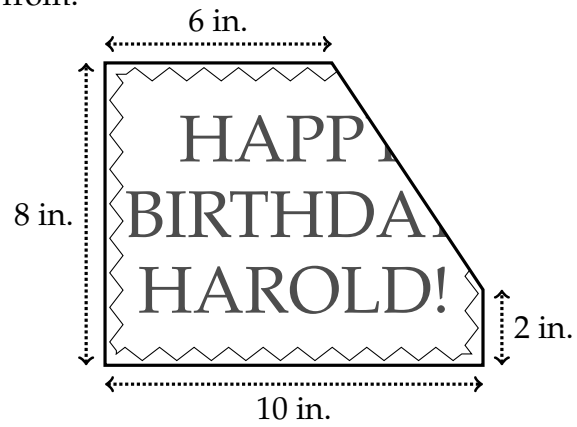
= 17.060 seconds

   (b) **[3 points]** While Electra is running through the puddle, how far does she run?

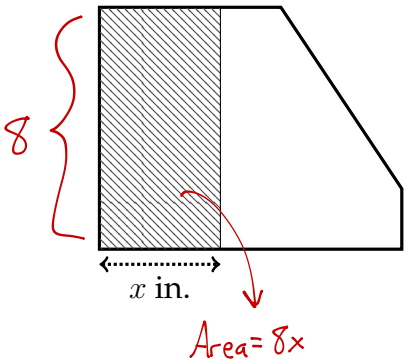
Distance = time * speed

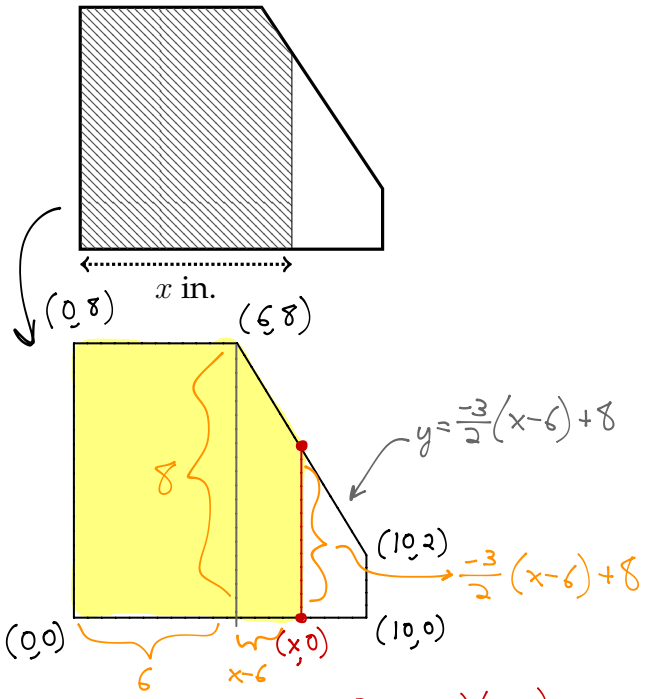
$= (17.060 \text{ sec.})(10 \text{ ft/sec}) =$ 170.6 ft

2. **[15 points]** Consider the following cake, which definitely isn't supposed to be a larger cake that I stole a piece from:



Suppose I make a cut x inches from the left end of the cake. Write a multipart function $f(x)$ for the area to the left of the cut, with domain $0 \leq x \leq 10$. Notice that there are two possible shapes I could get, depending on where I make the cut:





Area = $48 + \frac{1}{2}\left(8 + \frac{-3}{2}(x-6)+8\right)(x-6)$

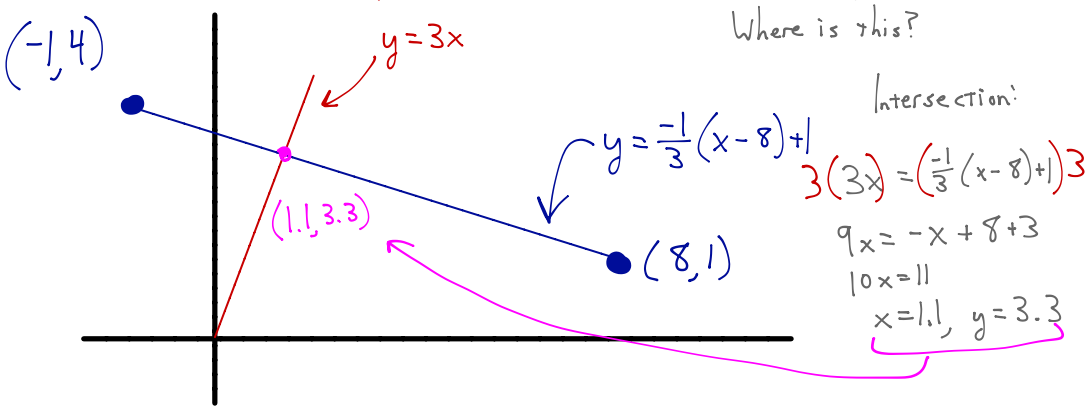
$= 48 + \frac{1}{2}\left(25 - \frac{3}{2}x\right)(x-6)$

$$f(x) = \begin{cases} 8x \text{ if } 0 \leq x \leq 6 \\ 48 + \frac{1}{2}\left(25 - \frac{3}{2}x\right)(x-6) \text{ if } 6 \leq x \leq 10 \end{cases}$$

3. Jareth stands at the point $(8, 1)$, and walks towards the point $(-1, 4)$ in a straight line at constant speed.

   Jareth reaches the point on his path closest to the origin 8 seconds into his journey.

   (a) **[6 points]** What is Jareth's speed, in units per second?



Intersection:

$3(3x) = \left(\frac{-1}{3}(x-8)+1\right)3$

$9x = -x + 8 + 3$

$10x = 11$

$x = 1.1,\ y = 3.3$

Jareth goes from $(8,1)$ to $(1.1, 3.3)$ in 8 seconds.

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{\sqrt{(8-1.1)^2 + (1-3.3)^2}}{8} \approx 0.909 \text{ units/sec}$$

   (b) **[5 points]** Give parametric equations for Jareth's coordinates after t seconds.

$x_0 = 8 \quad y_0 = 1$

$x_1 = 1.1 \quad y_1 = 3.3$

$\Delta x = -6.9 \quad \Delta y = 2.3$

$\Delta t = 8$

$$x = 8 - \frac{6.9}{8}t$$

$$y = 1 + \frac{2.3}{8}t$$

   (c) **[4 points]** When does Jareth cross the y-axis?

When his x-coordinate is 0: $8 - \frac{6.9}{8}t = 0$

$8 = \frac{6.9}{8}t$

$t = 9.275$ seconds

4. Phileas is going on a hot air balloon ride. His height above the ground after t minutes is a quadratic function of t.

   He begins from a platform at a height of 76 meters.

   The balloon reaches its maximum height after 30 minutes.

   After 80 minutes, he finally lands (at a height of 0 meters).

   (a) **[11 points]** Give a function $f(t)$ for Phileas's height above the ground after t minutes.

Seems like vertex form would help.

$f(t) = a(t-h)^2 + k$

$h = 30$

$f(0) = 76 \rightarrow 76 = a(0-30)^2 + k \rightarrow 76 = 900a + k$

$f(80) = 0 \rightarrow 0 = a(80-30)^2 + k \rightarrow -(0 = 2500a + k)$

$76 = -1600a$

$a = \frac{76}{-1600} = -0.0475$

$76 = 900(-0.0475) + k$

$\downarrow$

$k = 118.75$

$$f(t) = -0.0475(t-30)^2 + 118.75$$

   (b) **[4 points]** Find all times during Phileas's journey when he is at a height of exactly 100 meters.

$f(t) = -0.0475(t-30)^2 + 118.75 = 100$

$-0.0475(t-30)^2 = -18.75$

$(t-30)^2 = 394.737$

$t - 30 = \pm\sqrt{394.737}$

$t = 30 \pm \sqrt{394.737} =$ 10.13 and 49.87 minutes