Math 120 A - Winter 2015
Midterm Exam Number Two
February 26, 2015

Name: ______________________________ Student ID no. : ________________

Signature: ____________________________________ Section: ____________

1	15	
2	15	
3	15	
4	15	
Total	60	

- This exam consists of FOUR problems on FIVE pages, including this cover sheet.
- Show all work for full credit.
- You may use a scientific calculator during this exam. Graphing calculators are not permitted. Also, other electronic devices are not allowed, and should be turned off and put away for the duration of the exam.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- You may use one hand-written double-sided 8.5″ by 11″ page of notes.
- You have 50 minutes to complete the exam.

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1. **[15 points]** Rebecca is training to become a competitive eater. The time it takes her to eat a lobster is a linear-to-linear rational function of how long she spends practicing.

Without any practice, she can eat a lobster in 8 minutes.

If she practices for three weeks, she can finish it in 5 minutes and 6 seconds. → 5.1 min

If she practices for fifteen weeks, she can finish it in 3 minutes and 10 seconds. → $\frac{19}{6}$ min

How long should she practice in order to finish it in 2 minutes and 30 seconds?

$f(x)$ = time (in minutes) Rebecca takes after x weeks of practice

$$f(x) = \frac{ax+b}{x+d}$$

We know:

$f(0)=8 \Rightarrow \frac{b}{d}=8 \Rightarrow b=8d$

$f(3)=5.1 \Rightarrow \frac{3a+b}{3+d}=5.1 \Rightarrow 3a+b=15.3+5.1d \Rightarrow 3a+8d=15.3+5.1d$

$3a=15.3-2.9d$

$a=5.1-\frac{29d}{30}$

$f(15)=\frac{19}{6} \Rightarrow \frac{15a+b}{15+d}=\frac{19}{6} \Rightarrow 15a+b=47.5+\frac{19d}{6}$

$15\left(5.1-\frac{29d}{30}\right)+8d=47.5+\frac{19d}{6}$

$76.5-6.5d=47.5+\frac{19d}{6}$

$29=\frac{29d}{3}$

$\boxed{d=3}$

$b=8d \rightarrow \boxed{b=24}$

$a=5.1-\frac{29}{30}d \rightarrow \boxed{a=2.2}$

$$f(x)=\frac{2.2x+24}{x+3}$$

How long should she practice to get a time of 2.5 minutes?

$\frac{2.2x+24}{x+3}=2.5$

$2.2x+24=2.5x+7.5$

$16.5=0.3x$

$\boxed{x=55 \text{ weeks}}$

2. (a) **[8 points]** Let $f(x) = \log_2(x) + 6$.

Solve the equation $f(f(x)) = 9$.

$$f(f(x)) = 9$$
$$f(\log_2(x) + 6) = 9$$
$$\log_2(\log_2(x) + 6) + \underset{-6}{6} = \underset{-6}{9}$$

Recall: $\log_b c = a$ means that $c = b^a$ $\rightarrow$

$$\log_2(\log_2(x) + 6) = 3$$
$$\log_2(x) + \underset{-6}{6} = \underset{-6}{2^3}$$
$$\log_2(x) = 2 \rightarrow x = 2^2 \qquad \boxed{x = 4}$$

(b) **[7 points]** Let $f(x) = \log_2(x) + 6$, and let $g(x) = \ln(x)$.

Using scaling, shifting, and/or reflecting, explain how to transform the graph of $g(x)$ into the graph of $f(x)$.

Show your work below. You don't have to graph either function. (But you may, if it helps you visualize things.)

(Hint: Use some logarithm properties to rewrite $f(x)$.)

$$f(x) = \log_2(x) + 6$$
$$f(x) = \frac{\ln(x)}{\ln(2)} + 6 = \frac{g(x)}{\ln(2)} + 6$$

Write as: $y = \frac{g(x)}{\ln(2)} + 6$

$$\underbrace{\frac{y-6}{\frac{1}{\ln(2)}}}_{\text{this is the curve } y=g(x) \text{ after two transformations:}} = g(x)$$

Note: An alternate solution uses the fact that $6 = \log_2(64)$, so:

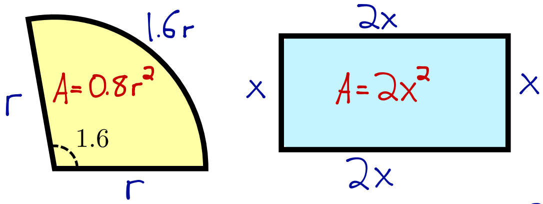
$$f(x) = \log_2(x) + \log_2(64)$$
$$= \log_2(64x)$$
$$= \frac{\ln\left(\frac{x}{1/64}\right)}{\ln(2)} \rightarrow g(x), \text{ scaled vertically and horizontally.}$$

Fill in the blanks:

- First, you scale vertically by a factor of $\frac{1}{\ln(2)}$.
- Then, you shift the graph up 6 units.

3. **[15 points]** You have 18 cm of wire that you will use to create two shapes: a sector with central angle 1.6 radians, and a rectangle that's twice as wide as it is long.

What should the radius of the sector be to **minimize** the combined area of the two shapes?



Optimize: Total area $= 0.8r^2 + 2x^2$

two variables!
Need some constraint.

Constraint: $2r + 1.6r + 6x = 18$

solve for x....

$6x = 18 - 3.6r$

$x = \underbrace{3 - 0.6r}$

plug into area

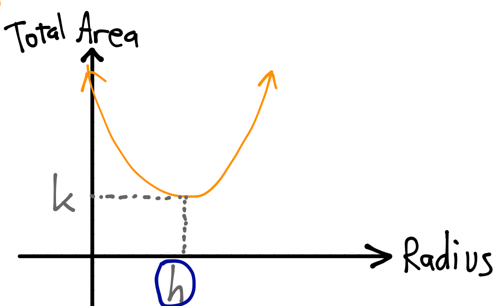
Total area $= 0.8r^2 + 2(3 - 0.6r)^2$

simplify

Total area $= 1.52r^2 - 7.2r + 18$

Upward-pointing quadratic.

Graph it?



Radius should be $h = \frac{-b}{2a} = \frac{7.2}{2 * (1.52)} = \boxed{2.368 \text{ cm}}$

4. (a) **[4 points]** The town of Beeattle has a **bee** problem.

In the year 2000, there were 5000 bees. Each year, the number of bees grows by 2.5%. Write a function $f(t)$ for the number of bees t years after the year 2000.

$A_0 = 5000 \qquad b = 1.025$

$$\boxed{f(t) = 5000(1.025)^t}$$

(b) **[6 points]** The town of Beeattle also has a **beetle** problem.

In the year 2010, there were 200 bees per beetle.
The beetle population doubles every 40 years.
Write a function $g(t)$ for the number of beetles t years after the year 2000.

$f(10) = 200g(10) \qquad b = 2^{1/40}$

$$5000(1.025)^{10} = 200 \cdot A_0\left(2^{1/40}\right)^{10}$$

$$32 = A_0\left(2^{1/40}\right)^{10}$$

$$A_0 = 26.91$$

$$\boxed{g(t) = 26.91\left(2^{1/40}\right)^t}$$

(c) **[5 points]** The town of Beeattle also has a **math** problem.

When will there be 300 bees per beetle? Round your answer to the nearest year.

$$5000(1.025)^t = 300 \cdot 26.91\left(2^{1/40}\right)^t$$

$$0.6193(1.025)^t = 2^{t/40}$$

$$\ln\left(0.6193(1.025)^t\right) = \ln\left(2^{t/40}\right)$$

$$\ln(0.6193) + t \cdot \ln(1.025) = t\,\frac{\ln(2)}{4}$$

$$t\left(\ln(1.025) - \frac{\ln(2)}{40}\right) = -\ln(0.6193)$$

$$t = \frac{-\ln(0.6193)}{\ln(1.025) - \frac{\ln(2)}{40}} \approx 65,$$ so around the year $\boxed{2065}$