# Math 120 A - Winter 2015
# Midterm Exam Number One
# January 29th, 2015

Name: _____ Student ID no. : _____

Signature: _____ Section: _____

| | | |
|---|---|---|
| 1 | 15 | |
| 2 | 15 | |
| 3 | 15 | |
| 4 | 15 | |
| Total | 60 | |

- This exam consists of FOUR problems on FIVE pages, including this cover sheet.
- Show all work for full credit.
- You may use a scientific calculator during this exam. Graphing calculators are not permitted. Also, other electronic devices are not allowed, and should be turned off and put away for the duration of the exam.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- You may use one hand-written double-sided 8.5" by 11" page of notes.
- You have 50 minutes to complete the exam.

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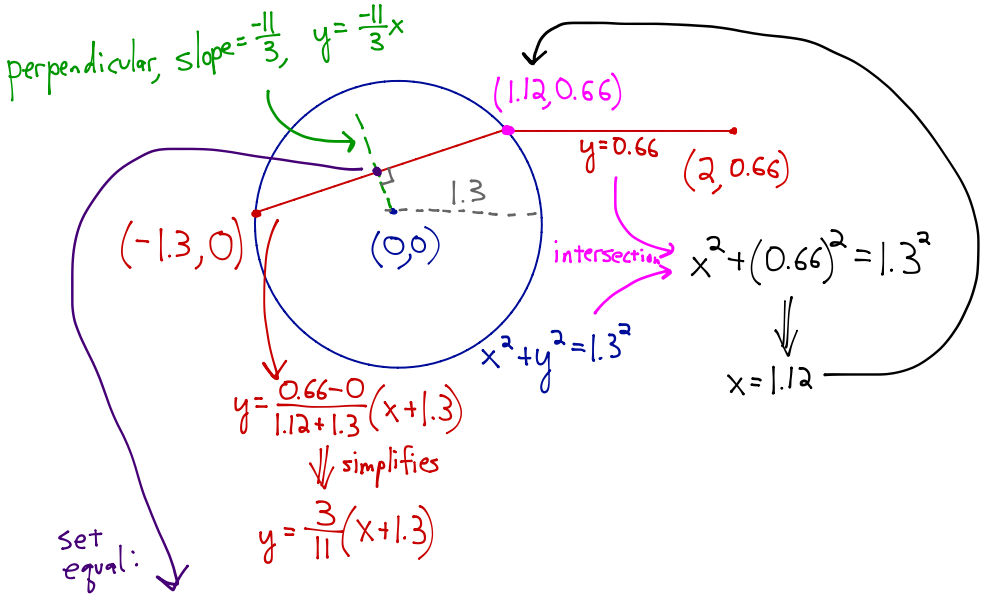
1. **[15 points]** Siddhi stands 2 kilometers east and 0.66 kilometers north of the **center** of a circular lake with radius 1.3 kilometers.

   First, she walks **due west** until she reaches the edge of the lake. Then swims in a straight line towards the **westernmost point** of the lake.

   How close does Siddhi come to the center of the lake?




set equal:

$$\frac{-11}{3}x=\frac{3}{11}(x+1.3)$$

$$\frac{-121}{9}x=x+1.3$$

$$\frac{-130}{9}x=1.3$$

$x=-0.09$, $y=\frac{-11}{3}x=0.33$

Closest point is $(-0.09, 0.33)$, so distance is $\sqrt{(-0.09)^2+(0.33)^2}$

$\approx \boxed{0.342 \text{ km}}$

2. Arthur is walking around the coordinate plane.

At time $t = 0$, he begins at the point $(5, -2)$. He walks in a straight line towards the point $(-7, 6)$ at a constant speed, reaching it in $8$ seconds.

(a) **[4 points]** Give parametric equations for Arthur's coordinates $t$ seconds after he begins walking.

$x_0 = 5$ $\quad y_0 = -2$

$x_1 = -7$ $\quad y_1 = 6$

$\Delta x = -12$ $\quad \Delta y = 8$

$\Delta t = 8$

$x = 5 + \frac{-12}{8}t$

$y = -2 + \frac{8}{8}t$ $\rightarrow$ $\boxed{x = 5 - \frac{3}{2}t \quad y = -2 + t}$

(b) **[6 points]** Find all times when Arthur is exactly $4$ units away from the origin.

$dist = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{(5 - \frac{3}{2}t)^2 + (-2+t)^2} = \sqrt{25 - 15t + \frac{9}{4}t^2 + 4 - 4t + t^2}$

$= \sqrt{\frac{13}{4}t^2 - 19t + 29} = 4$

$\frac{13}{4}t^2 - 19t + 29 = 16$

$\frac{13}{4}t^2 - 19t + 13 = 0$

$\downarrow$

$t = \frac{19 \pm \sqrt{(-19)^2 - 4(\frac{13}{4})(13)}}{2(\frac{13}{4})}$ or $\boxed{t \approx 0.791 \text{ and } 5.055 \text{ seconds}}$

(c) **[5 points]** Arthur drops his wallet $2$ seconds after crossing the $y$-axis. Where is it?

When does he cross the y-axis? $\rightarrow$ $x = 5 - \frac{3}{2}t = 0$

$t = \frac{10}{3}$ sec.

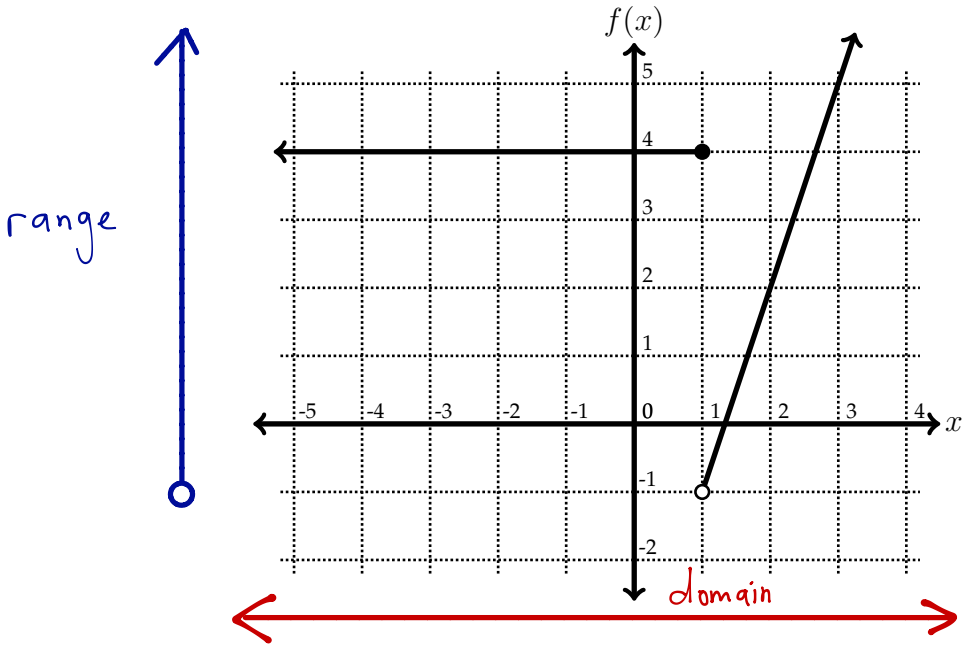
2 seconds later: $t = \frac{10}{3} + 2 = \frac{16}{3}$ seconds

Where is he then? $x = 5 - \frac{3}{2}(\frac{16}{3}) = -3$

$y = -2 + \frac{16}{3} = \frac{10}{3}$

So it's at $\boxed{(-3, \frac{10}{3})}$

3. Consider the function $f(x)$ shown in the graph below:




(a) **[4 points]** State the domain and range of $f(x)$.

Domain: $(-\infty, \infty)$ Range: $(-1, \infty)$

(b) **[5 points]** Write the multipart rule for $f(x)$.

$$f(x) = \begin{cases} 4 & \text{if } x \le 1 \\ 3x-4 & \text{if } x > 1 \end{cases}$$

(c) **[6 points]** Find all solutions to the equation $f(x) = 6 - x^2$

Set each part equal to $6-x^2$:

$x \le 1$: $4 = 6-x^2 \rightarrow x^2 = 2$, $x = \sqrt{2}$ or $-\sqrt{2}$

$\sqrt{2}$: not $\le 1$

$x > 1$: $3x-4 = 6-x^2 \rightarrow x^2+3x-10 \rightarrow (x+5)(x-2) = 0$

$x = -5$: not $> 1$

$x = 2$

two solutions

4. The temperature in the city of Pleasantville is a quadratic function of time.

   Today, the temperature is $70°$ Fahrenheit.

   $5$ days from now, the temperature will be $67°$ Fahrenheit.

   $10$ days from now, the temperature will be $68°$ Fahrenheit.

   (a) **[10 points]** Write a function $f(t)$ for the temperature $t$ days from now.

$f(t) = at^2 + bt + c$

$f(0) = 70 \rightarrow 0a + 0b + c = 70 \rightarrow c = 70$

$f(5) = 67 \rightarrow 25a + 5b + c = 67 \rightarrow 25a + 5b + 70 = 67 \rightarrow 25a + 5b = -3 \rightarrow b = \frac{-3}{5} - 5a$

$f(10) = 68 \rightarrow 100a + 10b + c = 68 \rightarrow 100a + 10b + 70 = 68 \rightarrow 100a + 10b = -2$

$100a + 10\left(\frac{-3}{5} - 5a\right) = -2$

$50a = 4$

$a = 0.08$

$b = \frac{-3}{5} - 5(0.08)$

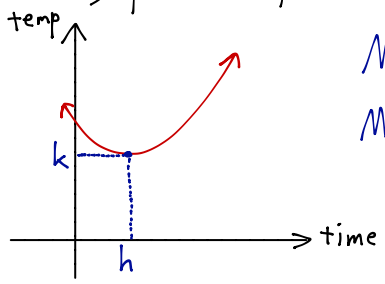
$b = -1$

$f(t) = 0.08t^2 - t + 70$

   (b) **[4 points]** When does Pleasantville reach its minimum temperature?

   What is that temperature?

$a > 0$, parabola points up:




Min. temp after $h = \frac{-b}{2a} = \frac{1}{2(0.08)} =$ 6.25 days

Min. temp is $k = c - \frac{b^2}{4a} = 70 - \frac{(-1)^2}{4(0.08)} =$ 66.875° F

   (c) **[1 point]** Your friends are thinking of visiting in 30 days. How will the weather be?

$f(30) = 0.08(30)^2 - (30) + 70 =$ 112°F, rather unpleasant