This exam consists of EIGHT problems on NINE pages, including this cover sheet.

- Show all work for full credit.
- You may use a scientific calculator during this exam. Graphing calculators are not permitted. Other electronic devices are not allowed, and should be turned off and put away for the duration of the exam.
- You do not need to simplify your answers.
- If you use a trial-and-error or guess-and-check method when a more rigorous method is available, you will not receive full credit.
- If you write on the back of the page, please indicate that you have done so!
- You may use one hand-written double-sided 8.5” by 11” page of notes.
- You have 170 minutes to complete the exam.
1. [12 points] In the following figure (not drawn to scale), find $x$. 

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[Diagram of a triangle with angles 33°, 40°, 50° and side 4]
2. The number of trees in Treeattle grows exponentially. Treeattle had 600 trees in the year 2008, and 1100 trees in the year 2015.

(a) [4 points] Write a function \( f(x) \) for the number of trees in Treeattle, \( x \) years after the year 2000.

(b) [6 points] Compute \( f^{-1}(x) \), the inverse of the function you found in part (a).

(c) [3 points] When will there be 4000 trees in Treeattle? Round your answer to the nearest year.
3.  (a) [3 points] Write a function \( f(x) \) for an upper semicircle of radius 4 centered at \((6, 2)\), defined over the interval \(2 \leq x \leq 10\).

(b) [3 points] Write a function \( g(x) \) for the curve obtained by taking \( f(x) \) from part (a), moving it 2 units to the left, and then scaling it horizontally by a factor of \(1/2\).

(c) [4 points] Find the domain and range of \( g(x) \).

(d) [3 points] Is \( g(x) \) one-to-one? Explain, briefly.
4. [9 points] In the following configuration, wheels A and B are connected by a belt, as are wheels C and D. Wheels B and C are connected by an axle.

Wheel A has a radius of 7 feet and rotates at a speed of 6 revolutions per minute.
Wheel B has a radius of 4 feet, Wheel C has a radius of 8 feet, and Wheel D has a radius of 3 feet.

How many seconds does it take Wheel D to make a complete rotation?
5. Tori and Harry are both running counter-clockwise around a circular track of radius 10 meters. Tori begins at the northernmost point and Harry begins at the easternmost point. Harry runs faster.

(a) [4 points] Tori first reaches the southernmost point after 8 seconds.

What is Tori’s speed, in meters per second?

(b) [6 points] Harry begins running at the same time as Tori, and catches up to her in 11 seconds.

What is Harry’s speed, in meters per second?

(c) [5 points] Impose a coordinate system with units in meters and the origin at the center of the circle. After 80 seconds, what are Harry’s coordinates?
6. Consider the following multipart function:

\[
f(x) = \begin{cases} 
  x^2 + 6x + 8 & \text{if } -4 \leq x < -1 \\
  3\sin\left(\frac{2\pi}{5}(x+1)\right) + 4 & \text{if } -1 \leq x < 9 
\end{cases}
\]

(a) [6 points] Sketch a graph of \( f(x) \). Label your graph clearly.

(b) [7 points] Find all solutions to the equation \( f(x) = 2 \).
7. Chloë and and Joë are walking around the coördinate plane. They both begin walking at the same time, in straight lines at constant speeds.

(a) [3 points] Chloë starts at \((-2, -3)\) and walks east at a speed of 4 units per second.
   Give parametric equations for Chloë’s coördinates after \(t\) seconds.

(b) [4 points] Joë begins at the point \((6, 3)\) and walks towards the point \((14, -5)\), reaching it in 4 seconds.
   Give parametric equations for Joë’s coördinates after \(t\) seconds.

(c) [5 points] When are Chloë and Joë closest together?
8. Let \( f(x) \) be the linear-to-linear rational function with an \( x \)-intercept of 5 and a \( y \)-intercept of \(-4\), passing through the point \((35, -6)\).

(a) [7 points] Write a formula for \( f(x) \).

(b) [2 points] Write the domain and range of \( f(x) \).

(c) [4 points] Solve the equation \( f(f(x)) = 2 \).