Math 120 (Pezzoli)
Winter 2014
Midterm #2

Name _____

TA: _____

Section: _____

Instructions:

- Your exam contains 4 problems.
- Your exam should contain 5 pages; please make sure you have a complete exam.
- Box in your final answer when appropriate.
- Unless stated otherwise, you **MUST** show work for credit. No credit for answers only. If in doubt, ask for clarification.
- Your work needs to be neat and legible.
- You are allowed one 8.5×11 sheet of notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed.
- Round off your answers to 2 decimal places, unless you are asked for exact answers.

Problem #1 (15 pts) _____

Problem #2 (15 pts) _____

Problem #3 (15 pts) _____

Problem #4 (15 pts) _____

TOTAL (60 pts) _____

1. Assume Tom invests $ 5000 at 2% annual interest compounded continually.

   (a) How much money will Tom have after 3 years ?

$$P(t) = 5000\, e^{0.02t}$$

$$P(3) = 5000\, e^{0.02 \cdot 3} \approx 5309$$

   (b) How long will it take for Tom's investment to double in value ?

$$10000 = 5000\, e^{0.02t}$$

$$2 = e^{0.02t}$$

$$\ln 2 = 0.02t$$

$$t = \frac{\ln 2}{0.02} \approx 34.66 \text{ years}$$

   (c) Assume that Bob also invested $5000 at annual interest r compounded continuously. Bob's investment doubled in 4 years. What is r ? Give the answer as a decimal.

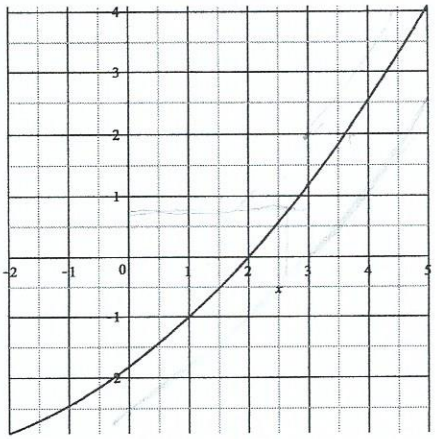
$$10\,000 = 5000\, e^{r \cdot 4}$$

$$2 = e^{r \cdot 4}$$

$$\ln 2 = r \cdot 4$$

$$r = \frac{\ln 2}{4} \approx 0.17$$

2. Below is the graph of the function $y = f(x)$ on the domain $-2 \leq x \leq 5$

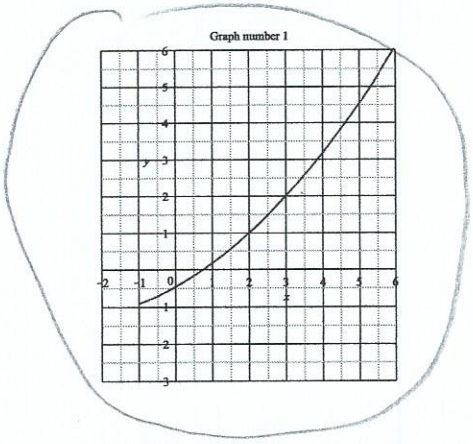


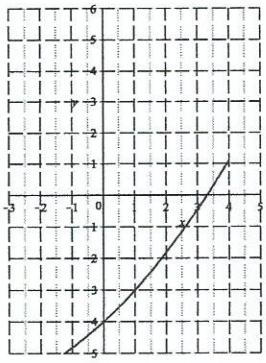
$f(x-1)$

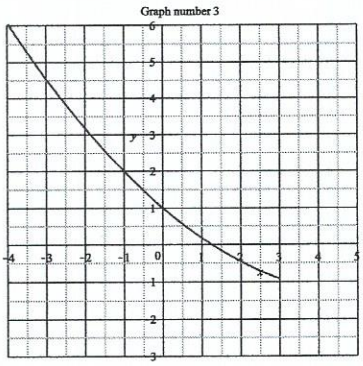
$-2 \leq x-1 \leq 5$

$-1 \leq x \leq 6$

(a) Which of the graphs below is the graph of $y = 2 + f(x-1)$? Circle the correct graph.







(b) If the domain of f is $-2 \leq x \leq 5$ what is the domain of the function $\frac{f(3x)+5}{x-1}$?

$x \neq 1$ $\quad -2 \leq 3x \leq 5 \quad -\frac{2}{3} \leq x \leq \frac{5}{3}$

$\left[-\frac{2}{3}, 1\right) \cup \left(1, \frac{5}{3}\right]$

(c) Compute $f^{-1}(-1)$

1

(d) If $h(x) = e^{f(x)}$ Which of the values below is closer to $h^{-1}(2)$? Circle the the right answer.

0.6 ,-1, 2.5, -2, 3.5

$2 = e^{f(x)}$

$\ln 2 = f(x)$

$0.69 \approx f(x)$

3. The population of Townsville was 20000 in the year 2000, 30,000 in 2005 and it is expected eventually to approach (but never exceed) 50,000. Measure time in years after 2000 and population in thousand of people. Find a linear to linear function P(t) that calculates the population of Townsville at time t.

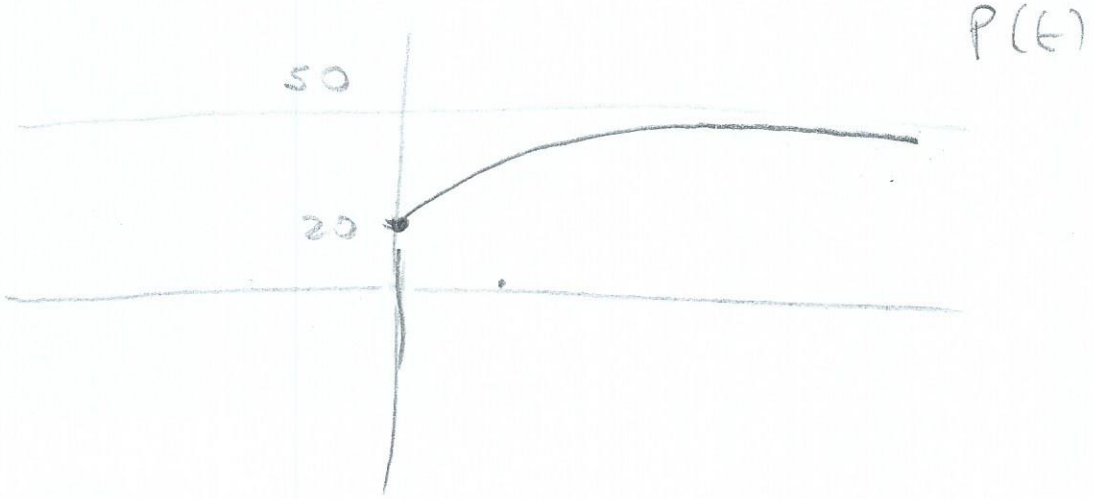
$$P(t) = \frac{at+b}{t+d}$$

$$\begin{cases} 20 = \frac{b}{d} & (t=0) \\ 30 = \frac{5a+b}{5+d} & (t=5) \\ a = 50 & (\text{horizontal asymptote } y=50) \end{cases}$$

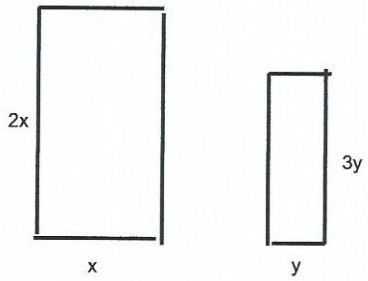
$$\begin{cases} a = 50 \\ b = 20d \\ 5\cdot 50 + 20d = 150 + 30d \end{cases}$$

$$\begin{cases} 10d = 100, \; d=10 \\ b = 200 \\ a = 50 \end{cases}$$

$$P(t) = \frac{50t+200}{t+10}$$



4. You have 300 m of fence to build two fencing enclosures as in the picure below: the first is a rectangle with one side twice as long as the other side. The second ia another rectangle with one side three times as long as the other one. For which value of x is the area of the two enclosure minimum ?



$A = 2x^2 + 3y^2$

$6x + 8y = 300 \qquad y = \frac{300 - 6x}{8}$

$A = 2x^2 + 3\left(\frac{300-6x}{8}\right)^2 =$

$= 2x^2 + \frac{3}{64} \cdot 36x^2 + \frac{3}{64}(-3600x) + \frac{3 \cdot 300^2}{64}$

The vertex of this pezebola $\cup$ has

x coordiele $-\frac{b}{2a} = \frac{\frac{3 \cdot 3600}{64}}{2\left(\frac{128 + 108}{64}\right)} =$

$= \frac{3 \cdot 3600}{64} \cdot \frac{64}{2 \cdot 236} = \frac{1350}{59} \simeq 22.88$

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Problem #2 (15 pts) _____

Problem #3 (15 pts) _____

Problem #4 (15 pts) _____

TOTAL (60 pts) _____

1. Assume Tom invests $ 5000 at 3% annual interest compounded continually.

(a) How much money will Tom have after 3 years ?

$P(t) = 5000\, e^{0.03t}$

$P(3) = 5000\, e^{0.09} \approx \$5470.87$

(b) How long will it take for Tom's investment to double in value ?

$10\,000 = 5000\, e^{0.03t}$

$2 = e^{0.03t}$

$\ln 2 = 0.03t$

$t = \frac{\ln 2}{0.03} \approx 23.1$ years

(c) Assume that Bob also invested $5000 at annual interest r compounded continuously. Bob's investment doubled in 5 years. What is r ? Give the answer as a decimal.

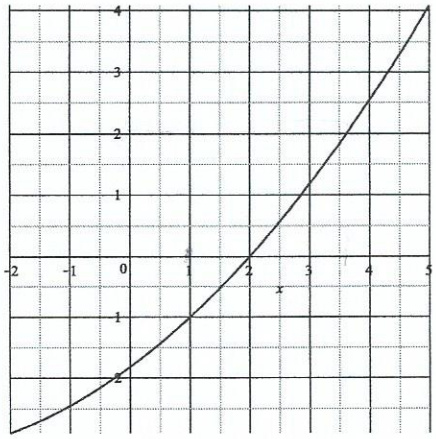
$10\,000 = 5000\, e^{r \cdot 5}$

$2 = e^{r5}$

$\ln 2 = 5r$

$r = \frac{\ln 2}{5} \approx 0.14$

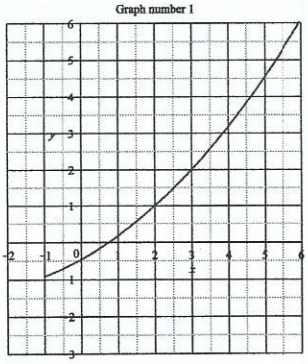
2. Below is the graph of the function $y = f(x)$ on the domain $-2 \leq x \leq 5$

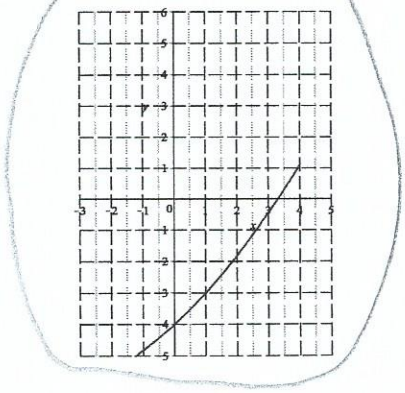


$-2 \leq x \leq 5$

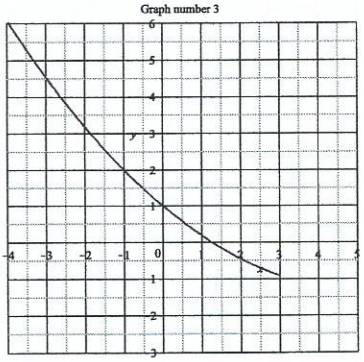
(a) Which of the graphs below is the graph of $y = f(x+1) - 3$? Circle the correct graph.

Graph number 1





Graph number 3



(b) If the domain of f is $-2 \leq x \leq 5$ what is the domain of the function $\frac{f(2x)+5}{x-2}$?

$x \neq 2$

$-2 \leq 2x \leq 5$

$-1 \leq x \leq \frac{5}{2}$

or $[-1, 2)$ and $(2, 2.5]$

(c) Compute $f^{-1}(-1)$

1

(d) If $h(x) = e^{f(x)}$ Which of the values below is closer to $h^{-1}(8)$? Circle the the right answer.

0.6 ,-1, 2.5, -2, 3.5

$8 = e^{f(x)}$ $\ln(8) = f(x)$ $2.08 = f(x)$

3. The population of Townsville was 20000 in the year 2000, 30,000 in 2005 and it is expected eventually to approach (but never exceed) 60,000. Measure time in years after 2000 and population in thousand of people. Find a linear to linear function P(t) that calculates the population of Townsville at time t, and sketch a graph of P(t) for $t \geq 0$

$$P(t) = \frac{at+b}{t+c}$$

$P(0) = 20$ $\quad 20 = b/c$

$P(5) = 30$ $\quad 30 = \frac{5a+b}{5+c}$

$P(t) \nearrow 60$ $\quad 60 = a$

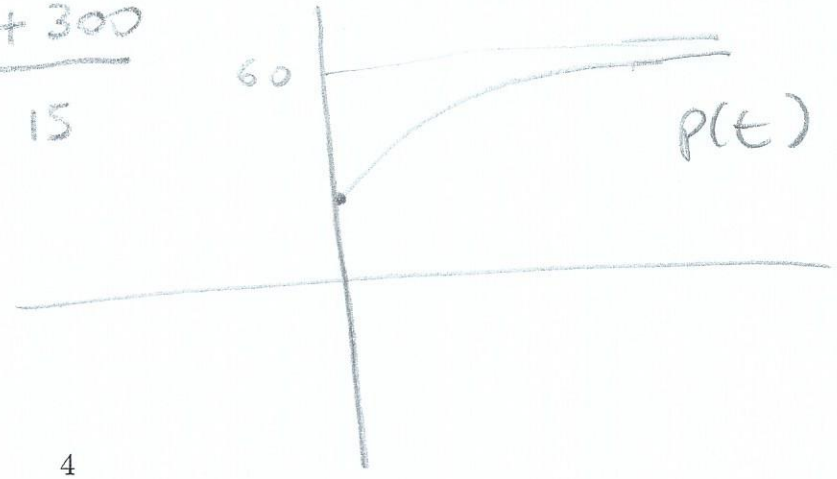
$$\begin{cases} a = 60 \\ b = 20c \\ 30 = \frac{5 \cdot 60 + 20c}{5+c} \end{cases}$$

$$\begin{cases} 30(5+c) = 300 + 60c \\ a = 60 \\ b = 20c \end{cases}$$

$$\begin{cases} 150 + 30c = 300 + 20c \\ a = 60 \\ b = 20c \end{cases}$$

$$\begin{cases} 10c = 150 \quad c = 15 \\ a = 60 \\ b = 300 \end{cases}$$

$$P(t) = \frac{60t + 300}{t + 15}$$



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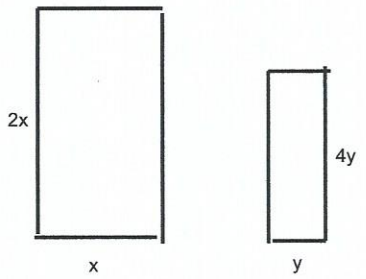

$A = 2x^2 + 4y^2$

$6x + 10y = 300$ $\qquad y = \frac{300 - 6x}{10}$

$A = 2x^2 + 4\left(\frac{300 - 6x}{10}\right)^2 =$

$= \frac{86}{25}x^2 - 144x + 3600$

$-\frac{b}{2a} = \frac{144 \cdot 25}{86} \simeq \boxed{20.93}$