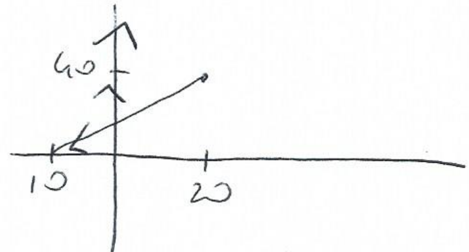
VERSION 1

1. At time $t = 0$ Tom leaves his house and starts driving North on a straight road at a constant speed of 30 mph. At the same time Bob leaves from a location situated 20 mi East and 40 mi North of Tom's house and starts driving on a straight road directly towards a location situated 10 mi West of Tom's house; Bob drives at a constant speed of 50 mph. Set up a coordinate system with the origin O corresponding to Tom's house, the x axis in the West East direction, the y axis in the South North direction.

   (a) Give Tom's coordinates at time t ($t \geq 0$).

$$\boxed{(0,\ 30t)}$$



   (b) Give Bob's coordinates at time t ($t \geq 0$). $(at+b,\ ct+d)$

at $t_1 = 0$ Bob at $(20, 40)$

at $t_2 = ?$ Bob at $(-10, 0)$ $\quad t_2 = \frac{d}{V} \quad d = \sqrt{(20-(-10))^2 + 40^2} = 50$

$t_2 = \frac{50}{50} = 1$

use $x = at + b$ $\qquad$ use $y = ct + d$

$20 = a \cdot 0 + b$ $\qquad$ $40 = c\,0 + d$

$-10 = a \cdot 1 + b$ $\qquad$ $0 = c\,1 + d$

$b = 20 \quad a = -30 \qquad d = 40 \quad c = -40$

$$\boxed{(-30t + 20,\ -40t + 40)}$$

   (c) When are Tom and Bob closest ?

$$d^2 = (-30t+20)^2 + (-40t+40-30t)^2$$

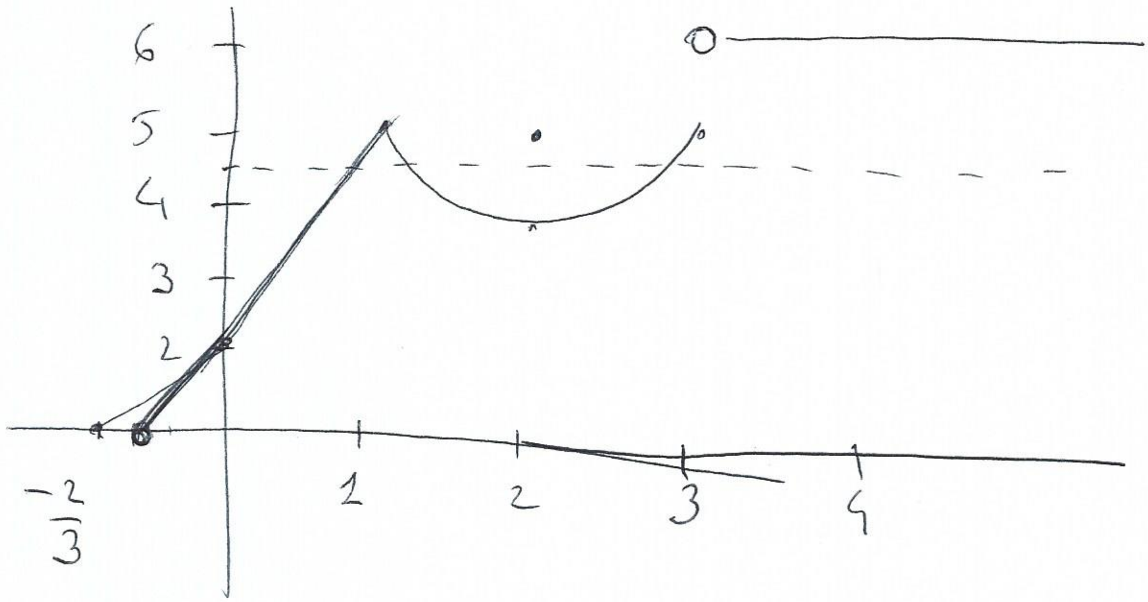
$$= 5800t^2 - 6800t + 2000 \qquad \text{parabola}$$

min is at vertex $\quad t = \frac{6800}{2 \cdot 5800} \approx \boxed{0.59 \text{ hrs}}$

2. Given the function

$$f(x) = \begin{cases} 3x+2, & \text{if } x \le 1 \\ 5 - \sqrt{1-(x-2)^2}, & \text{if } 1 < x \le 3 \\ 6 & \text{if } x > 3 \end{cases}$$

(a) Draw the graph of $y = f(x)$. Make sure to mark all relevant points on the axes.



(b) Find all solutions of the equation $f(x) = 4.5$

$3x + 2 = 4.5$ gives $x = \frac{2.5}{3} = \frac{5}{6} \approx \boxed{0.83}$

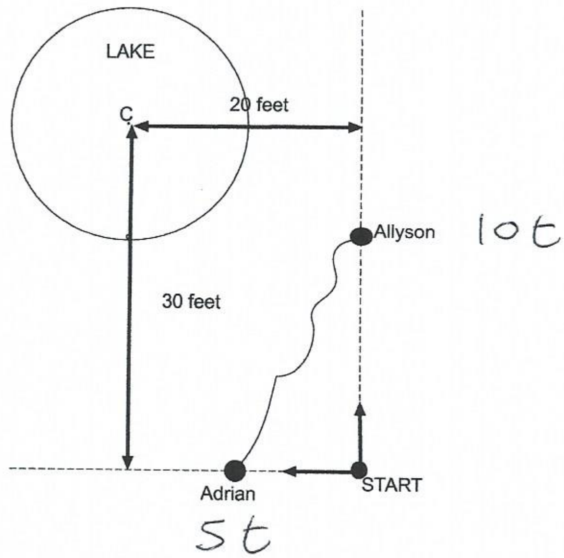
$5 - \sqrt{1-(x-2)^2} = 4.5$ gives

$0.5 = \sqrt{1-(x-2)^2}$ or $0.25 = 1 - (x-2)^2$

or $(x-2)^2 = \boxed{0.75}$ $x = 2 \pm \sqrt{0.75}$

$x \approx \boxed{1.13 \text{ or } 2.87}$

3. Allyson and Adryan have decided to connect their ankles with a bungee cord; one end is tied to each person ankle. The cord is 30 feet long, but it can stretch up to 90 feet . They both start at the same location. Allyson moves 10 ft/sec and Adrian moves 5 feet/sec in the directions indicated. Next to their starting location there is a lake that has the shape of a circle of radius 10 feet.



(a) Determine when the bungee cord first becomes tight (i.e. 30 feet long).

$$\sqrt{(5t)^2+(10t)^2} = 30 \qquad \sqrt{125}\,t = 30$$

$$t = \frac{30}{\sqrt{125}} \approx 2.68$$

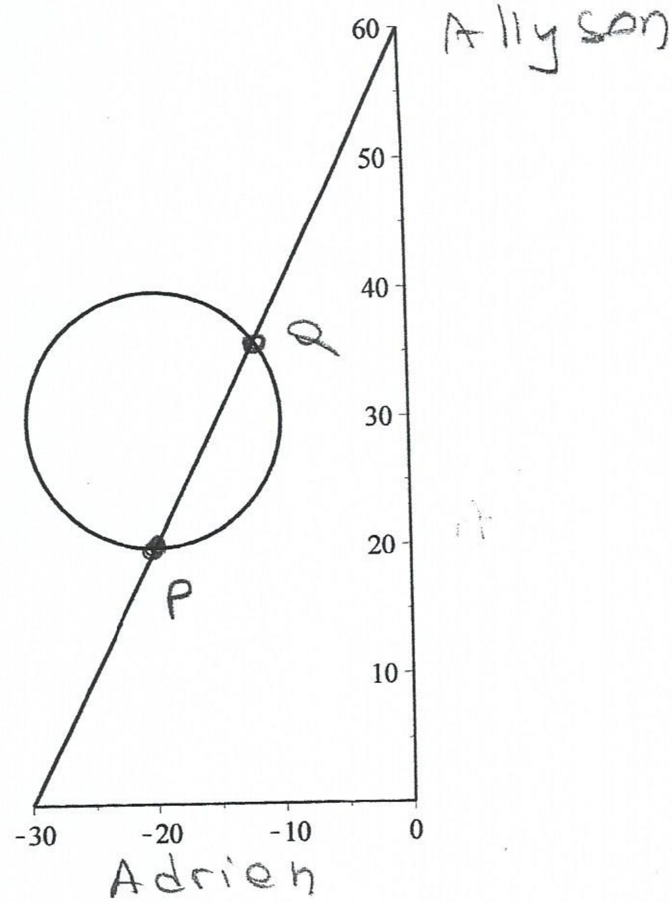
(b) At time $t = 6$ sec, how much of the bungee cord is over the water ?

At $t = 6$ Adrian is at $(-30, 0)$ Allyson at $(0, 60)$ bungee cord along the line

$$\frac{y-0}{x+30} = \frac{60-0}{30} \qquad y = 2x + 60 \qquad \text{Solve}$$

$$\begin{cases} (x+20)^2 + (y-30)^2 = 100 \\ y = 2x+60 \end{cases} \qquad \begin{cases} (x+20)^2 + (2x+60-30) = 100 \\ y = 2x+60 \end{cases}$$

$$\begin{cases} 5x^2 + 160x + 1200 \\ y = 2x+60 \end{cases} \qquad \begin{cases} x = -12, -20 \\ y = 36, 20 \end{cases}$$



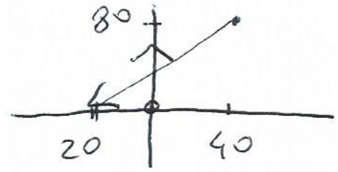
Q $(-12, 36)$ $\quad$ $P = (-20, 20)$

$d(P\ Q) = \sqrt{(-12+20)^2 + (36-20)^2} \approx$

$8\sqrt{5} \approx 17.89$ feet

VERSION 2

1. At time $t = 0$ Tom leaves his house and starts driving North on a straight road at a constant speed of 60 mph. At the same tine Bob leaves from a location situated 40 mi East and 80 mi North of Tom's house and starts driving on a straight road directly towards a location situated 20 mi West of Tom's house; Bob drives at a constant speed of 50 mph. Set up a coordinate system with the origin O corresponding to Tom's house, the x axis in the West East direction, the y axis in the South North direction.



(a) Give Tom's coordinates at time t ($t \geq 0$).

$$\boxed{(0,\ 60t)}$$

(b) Give Bob's coordinates at time t ($t \geq 0$). $(at+b,\ ct+d)$

At $t_1 = 0$ Bob at $(40, 80)$

at $t_2 = ?$ Bob at $(-20, 0)$ $\quad t_2 = \frac{d}{V}$

$d = \sqrt{(40-(-20))^2 + 80^2} = 100 \qquad t_2 = \frac{100}{50} = 2$ h

use $x = at+b$

$40 = 0 + b$

$-20 = 2a + b$

$b = 40 \quad a = -30$

use $y = ct+b$

$80 = 0 + d$

$0 = +2c + d$

$d = 80 \quad c = -40$

$$\boxed{(-30t + 40,\ -40t + 80)}$$

(c) When are Tom and Bob closest ?

$$d^2 = (-30t+40)^2 + (-40t+80-60t)^2$$

$$= 10900t^2 - 18400t + 8000$$

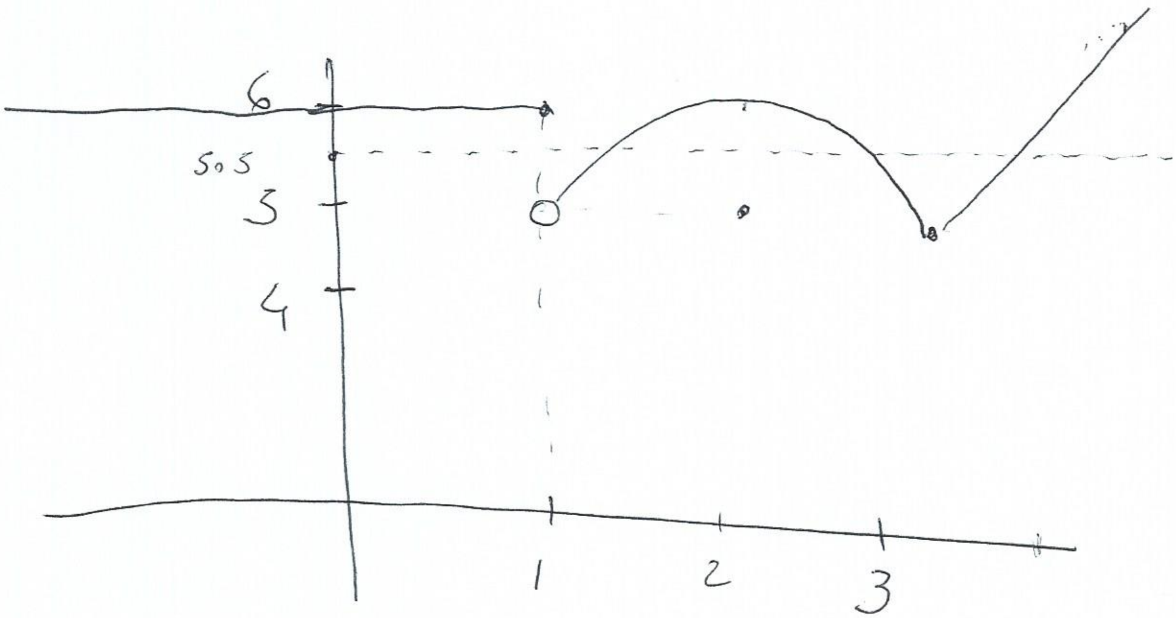
min is at vertex

$$t = \frac{18400}{2 \cdot 10900} \approx \boxed{0.84 \text{ hrs}}$$

2. Given the function

$$f(x) = \begin{cases} 6 & \text{if } x \le 1 \\ 5 + \sqrt{1 - (x-2)^2}, & \text{if } 1 < x \le 3 \\ 2x - 1 & \text{if } x > 3 \end{cases}$$

(a) Draw the graph of $y = f(x)$. Make sure to mark all relevant points on the axes.



(b) Find all solutions of the equation $f(x) = 5.5$

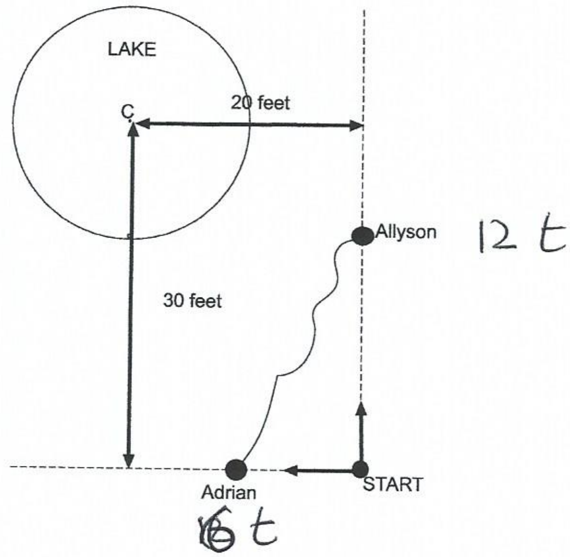
$5 + \sqrt{1-(x-2)^2} = 5.5$ or $\sqrt{1-(x-2)^2} = 0.5$ or

$1 - (x-2)^2 = 0.25$ or $0.75 = (x-2)^2$ or

$x = 2 \pm \sqrt{0.75}$ or $\boxed{x = 1.13 \quad x = 2.87}$

$2x - 1 = 5.5$ $\quad 2x = 6.5$ $\quad \boxed{x = 3.25}$

3. Allyson and Adryan have decided to connect their ankles with a bungee cord; one end is tied to each person ankle. The cord is 40 feet long, but it can stretch up to 90 feet . They both start at the same location. Allyson moves 12 ft/sec and Adrian moves 6 feet/sec in the directions indicated. Next to their starting location there is a lake that has the shape of a circle of radius 10 feet.



(a) Determine when the bungee cord first becomes tight (i.e. 40 feet long).

$$\sqrt{(12t)^2+(6t)^2} = 40$$

$$\sqrt{180}\ t = 40 \qquad t = \frac{40}{\sqrt{180}} \approx 2.98 \text{ sec}$$

(b) At time $t = 5$ sec, how much of the bungee cord is over the water ?

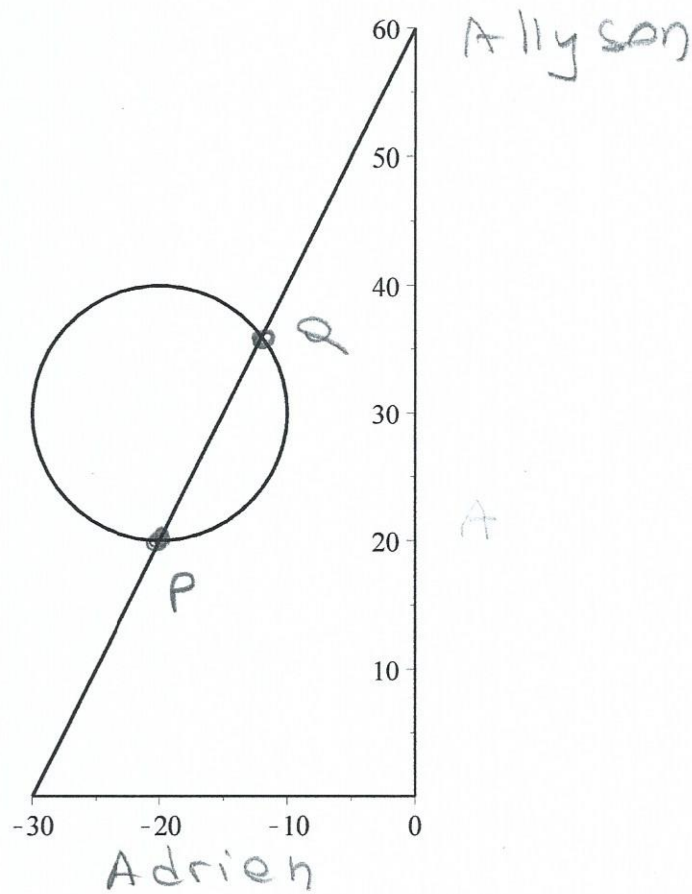
At $t = 5$ Adrien position is $(-30, 0)$

Allyson position is $(0, 60)$ bungee cord

along line $\frac{y-60}{x-0} = \frac{0-60}{-30-0}$ or $y = 2x+60$

solve $\begin{cases} (x+20)^2+(y-30)^2 = 100 \\ y = 2x+60 \end{cases}$ $\begin{cases} (x+20)^2+(2x+60-30)^2=100 \\ y=2x+60 \end{cases}$

$\begin{cases} 5x^2+160x+1200=0 \\ y = 2x+60 \end{cases}$ $\begin{cases} x = -12, -20 \\ y = 36, 20 \end{cases}$



$Q\ (-12, 36)$ $\quad$ $P = (-20, 20)$

$d(P\ Q) = \sqrt{(-12+20)^2 + (36-20)^2} \approx$

$8\sqrt{5} \approx 17.89$ feet