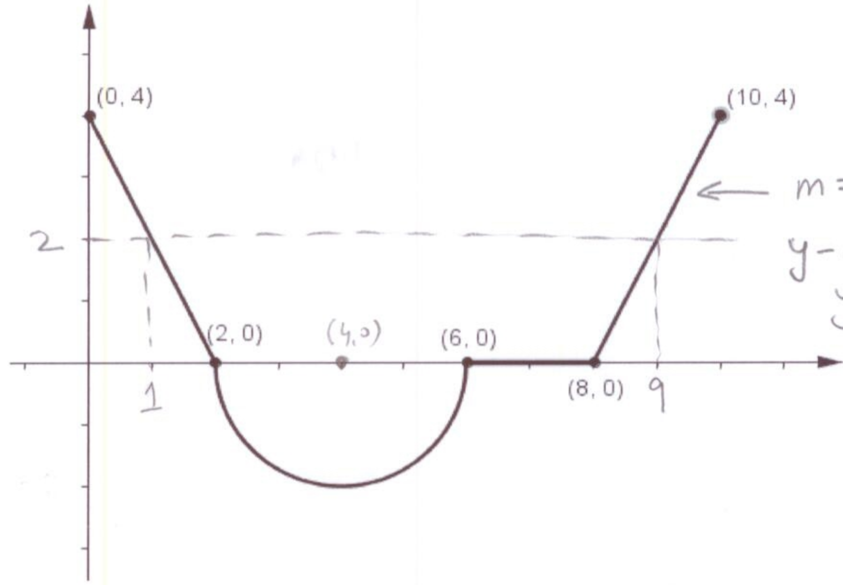
Math 120, Midterm 1, Winter 2014 $\boxed{\text{version 1}}$

1. (13 pts) The graph below shows a multi-part function, $y = f(x)$, consisting of 3 line segments and half of a circle. All questions below refer to this function. You do not have to show work in parts (a) and (b).



$m = \frac{4-0}{10-8} = \frac{4}{2} = 2$

$y - 0 = 2(x-8)$

$y = 2x - 16$

a) (8 pts) Write the multi-part rule for this function in terms of x. Include corresponding domains.

$$f(x) = \begin{cases} -2x+4 & \text{if } 0 \le x \le 2 \\ -\sqrt{4-(x-4)^2} & \text{if } 2 \le x \le 6 \\ 0 & \text{if } 6 \le x \le 8 \\ 2x-16 & \text{if } 8 \le x \le 10 \end{cases}$$

b) (2 pts) What is the range of this function? $\boxed{-2 \le y \le 4}$

c) (3 pts) Find all solutions to the equation $f(x) = 2$. You may use the graph, or solve algebraically, but justify how you get your answers.

The line $y=2$ crosses the graph of $f(x)$ at 2 points, where:

$-2x+4=2$ and where $2x-16=2$

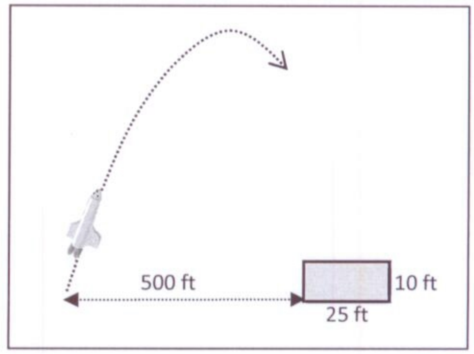
$-2x=-2$ $\qquad$ $2x=18$

$\boxed{x=1}$ $\qquad$ $\boxed{x=9}$

2. (13 pts) A rocket is launched as shown. The trajectory of the rocket is a parabola described by the equation

$$y = -0.01x^2 + 6x,$$

where y **is the height of the rocket above the ground**, and x **is the rocket's horizontal distance from its launching point** (origin), in feet. The rocket is meant to hit a flat-roofed storage building, 25 feet long and 10 feet tall, which is located 500 feet away from the rocket launcher, as shown in the diagram.



a) (5 pts) What is the maximum height of the rocket above the ground?

vertex: $x = \frac{-6}{2(-0.01)} = 300$ feet

max height $= y_{vertex} = -0.01(300)^2 + 6(300) = \boxed{900 \text{ feet}}$

b) (3 pts) Write down the rule $y = f(x)$ and the domain for the function describing just the **roof of the storage building.** No need to show work.

$\boxed{y = 10 \quad \text{if} \quad 500 \le x \le 525}$

c) (5 pts) Will the rocket hit the roof of the storage building?
If yes, find the coordinates (x, y) for the point of impact. If no, explain why not.

Rocket hits the building if $-0.01x^2 + 6x = 10$ for some x in $500 \le x \le 525$

Solving $-0.01x^2 + 6x - 10 = 0$

via Quadratic formula: $x = \frac{-6 \pm \sqrt{36 - 4(-0.01)(-10)}}{2(-0.01)}$

$= \frac{-6 \pm \sqrt{35.6}}{-0.02}$ → $1.671322\ldots$, $598.32867\ldots$ $\underbrace{}_{\text{not in domain}}$

So no, rocket does not hit the building

3. (14 pts) The traffic control tower at an airport has a circular radar range with a radius of 70 km around the tower. An airplane starts off at a point 80 km due North of the tower and flies in a straight line towards a point which is 100 km due East of the tower.

a) Draw a picture and impose a coordinate system with the origin at the tower. What is the x-coordinate of the airplane's position when it enters the range of the traffic control tower?

Path of plane has slope $m = \frac{0-80}{100-0} = -0.8$
and y-intercept 80, so
equation: $\boxed{y = -0.8x + 80}$

Radar circle: $\boxed{x^2 + y^2 = 4900}$

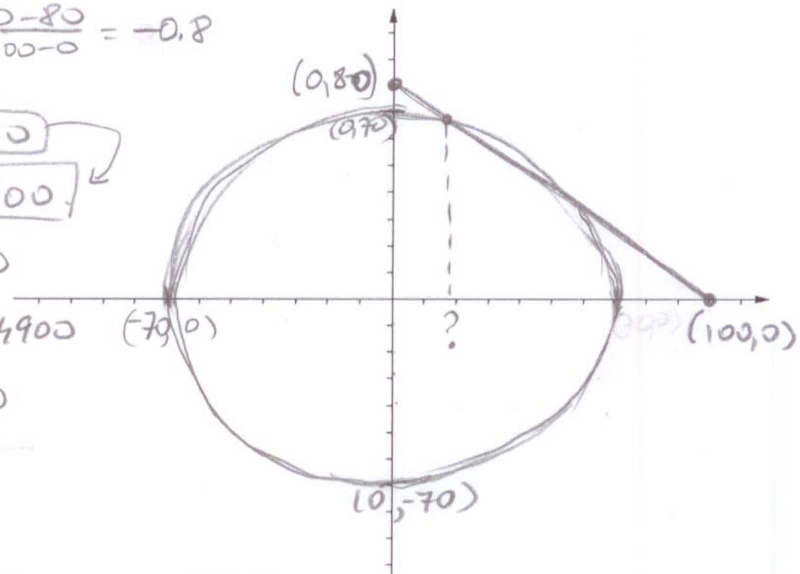
$x^2 + (-0.8x + 80)^2 = 4900$

$x^2 + 0.64x^2 - 128x + 6400 = 4900$

$1.64x^2 - 128x + 1500 = 0$

Q.F: $x = \frac{128 \pm \sqrt{6544}}{3.28}$

smaller x-value: $\frac{128 - 80.89\ldots}{3.28} \cong \boxed{14.36 \text{ km}}$



b) How close does the airplane get to the traffic control tower?

path: $y = -0.8x + 80$

perpendicular: $y = -\frac{1}{-0.8}x = 1.25x$

intersection point: $-0.8x + 80 = 1.25x$

$2.05x = 80$

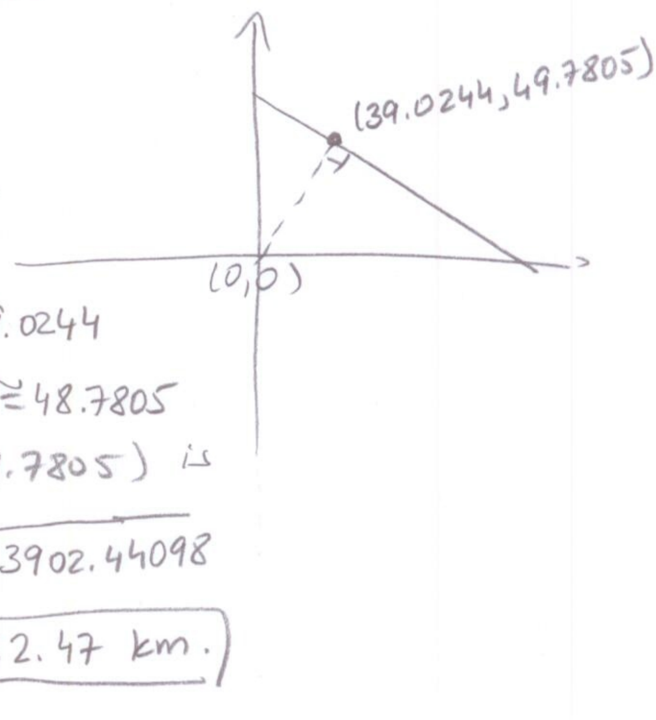
$x = \frac{80}{2.05} \cong 39.0244$

$y = 1.25(39.0244) \cong 48.7805$

Distance $(0,0)$ to $(39.0244, 48.7805)$ is

$d \cong \sqrt{(39.0244)^2 + (48.7805)^2} = \sqrt{3902.44098}$

$\cong \boxed{62.47 \text{ km.}}$



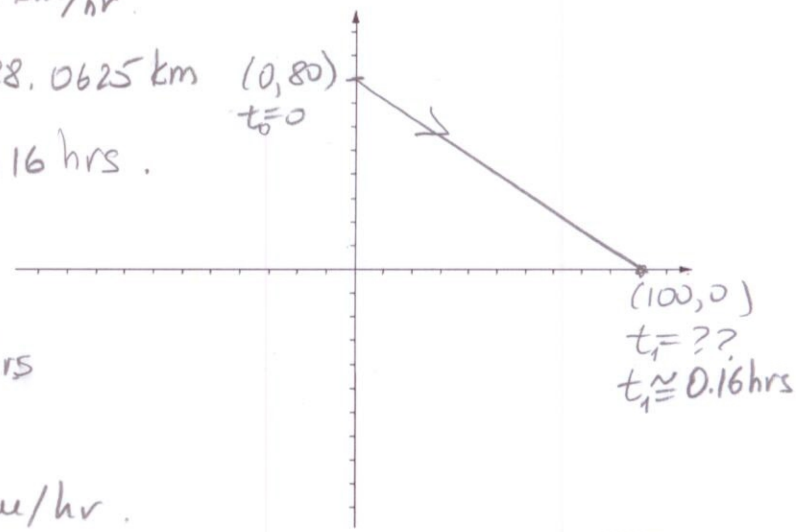
4. (10 pts) Consider the same plane as in the previous problem: the airplane starts off at a point 80 km due North of the tower and flies in a straight line towards a point which is 100 km due East of the tower. Suppose the plane flies at a speed of 500 miles per hour ($1\ mile \approx 1.6\ km$).

Compute the parametric equations for the plane, and determine the position of the airplane six minutes after it starts flying.

Speed $\cong 500 \frac{\cancel{miles}}{hr} \times \frac{1.6\ km}{1\ \cancel{mile}} = 800\ km/hr$

distance $= \sqrt{80^2+100^2} = \sqrt{16{,}400} \cong 128.0625\ km$

time $= \frac{dist.}{speed} = \frac{128.0625}{800} \cong 0.16$ hrs.



Horizontal velocity is

$v_x \cong \frac{100-0}{0.16} = 625$ km/hrs

Vertical velocity is:

$v_y \cong \frac{0-80}{0.16} = -500$ km/hr.

Parametric equations: $\begin{array}{l} x = x_0 + v_x t \\ y = y_0 + v_y t \end{array} \Rightarrow \boxed{\begin{array}{l} x = 625t \\ y = 80 - 500t \end{array}}$

x, y in kilometers
t in hours.

After 6 minutes $= \frac{6}{60}$ hours $= 0.1$ hrs:

$x = 625(0.1) = 62.5$

$y = 80 - 500(0.1) = 30$

so the position of the plane is: $\boxed{(62.5, 30)}$ (in kilometers)

(i.e. 62.5 km East and 30 km North of the Tower)