NAME: ______________________________ Student ID #: ________________

QUIZ SECTION:______

Math 120A
Midterm II
February 20th, 2014

Problem 1	12	
Problem 2	10	
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Total:	50	

- Please do not open your exam until instructed to do so.
- You are allowed to use a non-graphing calculator and one double-sided handwritten sheet of notes Do not share notes.
- When the exam starts, **check that you have a complete exam**. In addition to this cover page, there should be **4 problems on 4 pages**.
- Unless otherwise stated, you **MUST SHOW YOUR WORK**. Correct (or incorrect) answers with no supporting work may result in little or no credit.
- Make sure to read each question carefully, and not to spend too much time on any one page. Aim for about 12 min/page.
- Please box your **final answer** to each question.
- You may round off your final answers to 2 decimal digits. Do not round bases of exponential functions, and keep at least 4 digits of precision throughout the rest of your computations.
- If you need more room, use the backs of pages and indicate to the grader that you have done so. There is also an extra page at the end of the exam, for scratch work, if needed.
- Raise your hand if you have a question.

GOOD LUCK!

1. (12 pts)

a) (8 pts) Suppose you have some function $y = f(x)$ such that:

- the domain of $f(x)$ is: $1 \leq x \leq 5$ and
- the range of $f(x)$ is: $-2 \leq f(x) \leq 6$.

i. Compute the **domain** of $g(x) = 3f(2x+1) + 4$.

Domain of $f(x)$ is $1 \leq x \leq 5$

so we must have $1 \leq 2x+1 \leq 5$ for $f(2x+1)$

Subtract 1: $0 \leq 2x \leq 4$

Divide by 2: $\boxed{0 \leq x \leq 2}$ i.e. $\boxed{[0,2]}$

ii. Compute the **range** of $g(x) = 3f(2x+1) + 4$.

Range of $f(2x+1)$ is the same as range of $f(x)$:

$-2 \leq f(2x+1) \leq 6$

$\times 3$

$-6 \leq 3f(2x+1) \leq 18$

$+4$

$\boxed{-2 \leq 3f(2x+1)+4 \leq 22}$

i.e. $\boxed{[-2, 22]}$

b) (4 pts) Solve the following equation for x. Show your steps.

$$\log_3(\ln(x)) = 2$$

$$3^{\log_3(\ln(x))} = 3^2$$

$$\ln(x) = 9$$

$$\boxed{x = e^9}$$

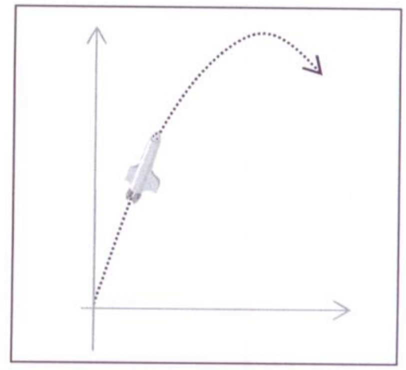
$$\boxed{\cong 8103.08}$$

2. (10 pts)

The trajectory of a rocket is a parabola described by the function

$$y = -0.01x^2 + 6x,$$

where y is the height of the rocket above the ground, and x is the rocket's horizontal distance from its launching point, in feet.



a) (2 pts) Compute the coordinates of the vertex of this function.

$$x = -\frac{b}{2a} = \frac{-6}{2(-0.01)} = 300$$

$$y = -0.01(300)^2 + 6(300) = 900$$

so vertex is at $\boxed{(300, 900)}$

b) (8 pts) Compute a function $x = g(y)$ relating the x-coordinate of the rocket to its height y, **while the rocket is going down** (past the vertex). Also, specify the domain and range of this function.

We need to find the inverse function of

$y = -0.01x^2 + 6x$ restricted to $x \geq 300$

(so: domain $x \geq 300$
range $y \leq 900$)

$$-0.01x^2 + 6x - y = 0$$

$$0.01x^2 - 6x + y = 0$$

Q.F. $x = \dfrac{6 \pm \sqrt{36 - 4(0.01)y}}{0.02}$

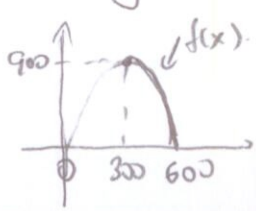
$= 300 \pm 50\sqrt{36 - 0.04y}$

Since $x \geq 300$

$\Rightarrow \boxed{x = 300 + 50\sqrt{36 - 0.04y}}$

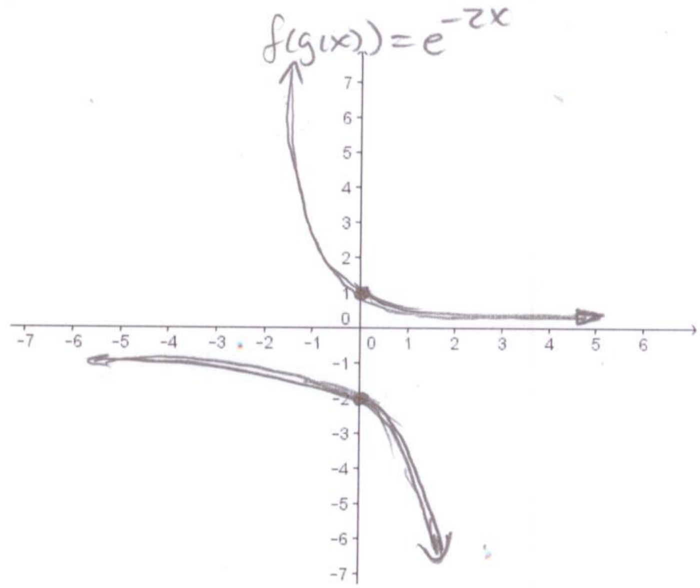
$\left\{ \begin{array}{l} \text{Domain: } y \leq 900 \\ \text{Range: } x \geq 300 \end{array} \right.$

[Technically we also need $y \geq 0$ so domain is $[0, 900]$ and range is $[300, 600]$]



3. (14 pts) In this problem, you need not show work or justify your answers (other than what is explicitly asked).

a) (6 pts) **Let $f(x) = e^x$ and $g(x) = -2x$.**

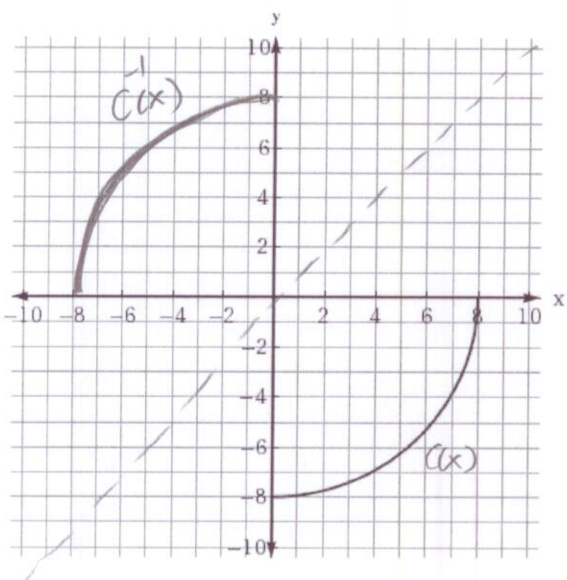


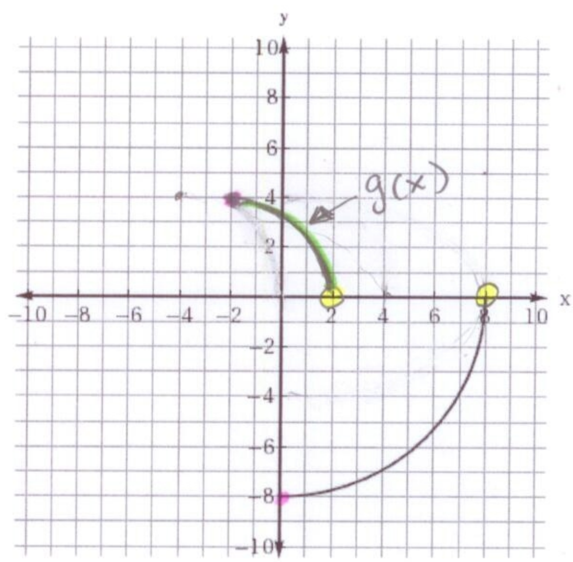
Compute $f(g(x))$= e^{-2x}

Compute $g(f(x))$= $-2e^{x}$

Sketch the graphs of $f(g(x))$ and $g(f(x))$
(correct shape and y-intercept),
and label each one.

c) (8 pts) **Consider the function $y = C(x)$ whose graph is shown below, twice.**





i. (2 pts) Sketch the graph of the inverse function, $C^{-1}(x)$, on the leftmost graph provided.

ii. (6 pts) Sketch the graph of $g(x) = -\frac{1}{2}C(2x+4)$ on the rightmost graph provided

and list the graph operations performed, explicitly and in correct order:

Vertically: First compress $\times \frac{1}{2}$ then Reflect in x-axis (or vice-versa)

Horizontally: First SHIFT LEFT 4 units then compress $\times \frac{1}{2}$

(for instance: First: "shift up by 3 units", etc)

1 $(0,-8) \xrightarrow{y/2} (0,-4) \xrightarrow{-y} (0,4) \xrightarrow{x-4} (-4,4) \xrightarrow{x/2} (-2,4)$

2 $(8,0) \longrightarrow (8,0) \longrightarrow (8,0) \longrightarrow (4,0) \longrightarrow (2,0)$

4. (14 pts) The town of Metropolis is suffering from a serious infestation with Zombies and Werewolves. At 12:00am on Halloween, there were 27 Zombies in Metropolis. The number of Zombies grows exponentially, doubling every two hours.

a) Find a function $Z(x)$ modeling the number of Zombies in Metropolis x hours after midnight. Show work.

$$Z(x) = Z_0 b^x$$

$Z(0) = 27 \Rightarrow Z(x) = 27 b^x$

$Z(2) = 54 \Rightarrow Z(2) = 27 b^2 = 54 \Rightarrow b^2 = 2 \Rightarrow b = \sqrt{2}$

$\boxed{Z(x) = 27(\sqrt{2})^x}$ $x = \#$ hours since 12:00am.

b) The Werewolf population also grows exponentially. At 8:00am on Halloween there were 576 Werewolves in Metropolis. At noon that same day, there were equal numbers of Zombies and Werewolves. Find a function $W(x)$ modeling the number of Werewolves in Metropolis x hours after midnight.

$$W(x) = W_0 d^x$$

noon: $x = 12$

$Z(12) = 27(\sqrt{2})^{12}$

$= 27(2^6)$

$= 1728$.

2 data points:

1) 8am: $(8, 576) \Rightarrow W_0 d^8 = 576$

2) noon: $(12, 1728) \Rightarrow W_0 d^{12} = 1728$

$\frac{2^{nd}}{1^{st}}$: $\frac{\cancel{W_0} d^{12}}{\cancel{W_0} d^8} = \frac{1728}{576}$

$d^4 = 3$

$d = \sqrt[4]{3} \Rightarrow W_0 = \frac{576}{(\sqrt[4]{3})^8} = \frac{576}{9} = 64$

So: $\boxed{W(x) = 64(\sqrt[4]{3})^x}$

c) At what time will there be twice as many Zombies as Werewolves in Metropolis? Round your answer to the nearest minute (ex: 16:07, or 4:07pm).

$$\frac{Z(x)}{W(x)} = 2 \Rightarrow \frac{27(\sqrt{2})^x}{64(\sqrt[4]{3})^x} = 2$$

$$\left(\frac{\sqrt{2}}{\sqrt[4]{3}}\right)^x = \frac{128}{27}$$

$$x \ln\left(\frac{\sqrt{2}}{\sqrt[4]{3}}\right) = \ln\left(\frac{128}{27}\right)$$

$$x = \frac{\ln\left(\frac{128}{27}\right)}{\ln\left(\frac{\sqrt{2}}{\sqrt[4]{3}}\right)} \cong 21.6376\ldots \Rightarrow \boxed{21:38}$$

or $\boxed{9:38\text{pm}}$