Math 120 A, B - Winter 2009
Final Exam
March 14, 2009
Answers

1. 87.0494546 feet
2. 26.61 seconds.
3. 17.9507 years after 2000.
4. 6.184685 hours
5. (a) $x = -5 + 3t$, $y = 5 - t$ (b) $x = t$, $y = 9 - 3t$ (c) 2.25 seconds
6. $$D(t) = \begin{cases} 4t & \text{if } 0 \le t \le \frac{1}{4}; \\ \sqrt{1 + (4(t - \frac{1}{4}))^2} & \text{if } \frac{1}{4} < t < \frac{3}{4}; \\ \sqrt{2^2 + (1 - 4(t - \frac{3}{4}))^2} & \text{if } \frac{3}{4} < t < \frac{3}{2} \end{cases}$$
7. (a) We have
$$f(g(x)) = f(3x-1) = 3x - 1 + 2\begin{cases} 3x-4 & \text{if } 3x-4 \ge 0 \\ 4-3x & \text{if } 3x-4 < 0 \end{cases} = \begin{cases} 9x-9 & \text{if } x \ge 4/3 \\ 7-3x & \text{if } x < 4/3 \end{cases}$$
To solve $f(g(x)) = -4x$ we must solve two equations
$$9x - 9 = -4x \text{ subject to } x \ge 4/3$$
and
$$7 - 3x = -4x \text{ subject to } x < 4/3.$$
The first equation yields $x = 9/13$ which is not greater than $4/3$, so it is not a solution.
The second yields $x = -7$ which is less than $4/3$, so it is a solution.
Thus the only solution is $x = -7$.
(b) We have
$$h(x) = g(\sqrt{x}) + x + 1 = 3\sqrt{x} - 1 + x + 1 = 3\sqrt{x} + x.$$
Setting $h(x) = y$ and solving for x we have
$$3\sqrt{x} + x = y$$
$$3\sqrt{x} = y - x$$
$$9x = (y-x)^2 = y^2 - 2xy + x^2$$
$$x^2 - 2xy - 9x + y^2 = 0$$
$$x^2 - (2y+9)x + y^2 = 0$$
and so
$$x = \frac{2y + 9 \pm \sqrt{36y + 81}}{2} = y + \frac{9}{2} \pm \sqrt{9y + \frac{81}{4}}.$$

Because of the $\pm$, we much choose one or the other to get the inverse function. A good way to do this is to use any value of h. For instance, we see that $h(0) = 0$. Hence, $h^{-1}(0)$ must be zero as well. If we substitute zero for y in the last expression, we find

$$x = 0 + \frac{9}{2} \pm \sqrt{\frac{81}{4}}$$

which equals zero only if we take the "$-$" option. Thus the inverse function is

$$h^{-1}(x) = x + \frac{9}{2} - \sqrt{9x + \frac{81}{4}}.$$