Math 120 - Winter 2006
Exam 2
February 23, 2006

Name: Instructor's Key

Section: Version 1

Student ID Number: ______________________

TA's Name: ______________________

1	10	
2	10	
3	10	
4	10	
5	10	
Total	50	

- You are allowed to use a calculator and one **hand-written** 8.5 by 11 inch page of notes. Put your name on your sheet of notes and turn it in with the exam.
- Check that your exam contains all the problems listed above.
- You must show your work on all problems. The correct answer with no supporting work may result in no credit. Unless otherwise indicated, your **final answer** must be correct to two digits after the decimal.
- If you need more room, use the backs of the pages and indicate to the grader that you have done so.
- Raise your hand if you have a question.
- There are multiple versions of the exam. Any student found engaging in academic misconduct will receive a score of 0 on this exam.
- You have 50 minutes to complete the exam.

GOOD LUCK!

1. (10 points) Dr. Loveless is getting ready for a tennis tournament. The speed of his serve, y, is a linear-to-linear function of the number of practice balls, x, he hits before the tournament. If he hits no practice balls, then the speed is 85 mph. If he hits 100 practice balls, then the speed is 105 mph. Dr. Loveless knows from his younger days that if he keeps increasing the number of practice balls, his serve speed increases and approaches (but never exceeds) 130 mph.

How many practice serves should Dr. Loveless hit so that his serve speed is exactly 125 mph?

+2 $y = \dfrac{ax+b}{x+d}$ $\quad a = 130 \quad (0,85) \quad (100,105)$

+2 $y = \dfrac{130x+b}{x+d}$

+1 $85 = \dfrac{130(0)+b}{(0)+d} \Rightarrow \boxed{b = 85d}$

+1 $105 = \dfrac{13000+b}{100+d} \Rightarrow \boxed{10500+105d = 13000+b}$

$10500 + 105d = 13000 + 85d$

$20d = 2500$

+1 $\boxed{d = 125}$

$\boxed{b = 85(125) = 10625}$

$\boxed{y = \dfrac{130x+10625}{x+125}}$

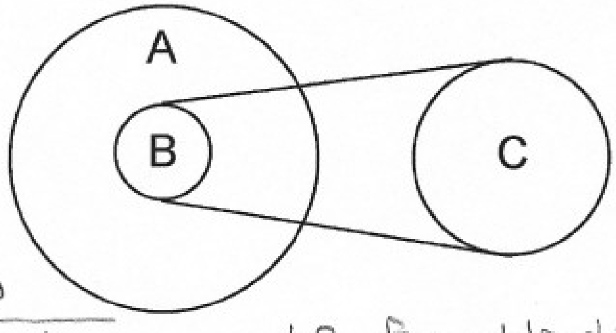
+2 $125 = \dfrac{130x+10625}{x+125}$

$125x + 15625 = 130x + 10625$

+1 $5000 = 5x$

$\boxed{x = 1000 \text{ balls}}$

2. (10 points) Consider the following belt and wheel system with the three wheels A, B, and C. Notice that wheels A and B share the same axis and wheels B and C are connected by a belt. Wheel A has radius 10 inches and wheel B has radius 4 inches. The linear speed of wheel A is 30 inches per second and the angular speed of wheel C is 0.3 revolutions per second. What is the radius of wheel C?



	v	ω
$r=10\text{in}$ A	$30\ \frac{in}{sec}$	$\boxed{I}$ $3\ \frac{rad}{sec}$
$r=4\text{in}$ B	$\boxed{III}$ $12\ \frac{in}{sec}$	$\boxed{II}$ $3\ \frac{rad}{sec}$
$r=???$ C	$\boxed{IV}$ $12\ \frac{in}{sec}$	$0.6\pi\ \frac{rad}{sec}$

+2 for relationships

+2

$0.3\ \frac{rev}{sec} = \frac{0.6\pi\ rad}{sec}$

$= 1.88495559215\ \frac{rad}{sec}$

$V = \omega r \qquad \omega = \frac{V}{r} \qquad r = \frac{V}{\omega}$

+2 $\boxed{I}$ $\omega = \frac{30}{10} = 3\ \frac{rad}{sec}$

$\boxed{II} = \boxed{I}$

$\boxed{III}$ $v = \omega r = 3 \cdot 4 = 12\ \frac{in}{sec}$

+2 $\boxed{IV} = \boxed{III}$

This does not have to be written out, but it gives an explanation for what is happening in the table.

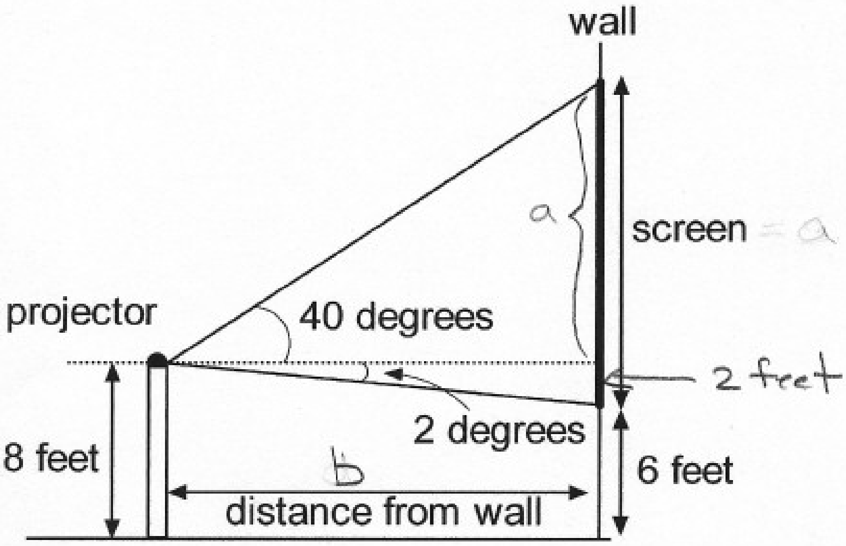
$r = \frac{V}{\omega} = \frac{12}{0.6\pi} \approx 6.36619772368$ inches

+2

$\boxed{6.37 \text{ inches}}$

3. (10 points) Larry needs to buy and install a giant projector screen on a wall. The projecting machine will sit on a podium so that it is 8 feet above the ground. The projector outputs light downward from horizontal at 2 degrees and upward from horizontal at 40 degrees. The screen must be installed so that the bottom is exactly 6 feet above the ground. This situation (viewed from the side) is illustrated below.

Find the distance that the projector should be placed away from the wall and the height of the screen so that the projected light will exactly cover the screen.



+4 $\tan(2^\circ) = \frac{2}{b} \Rightarrow b = \frac{2}{\tan(2^\circ)} \approx 57.272506566$

+4 $\tan(40^\circ) = \frac{a}{b} = \frac{a}{57.272506566}$

+1 $a = b\tan(40^\circ) \approx 48.057339136$

+1 distance from wall = 57.27 feet

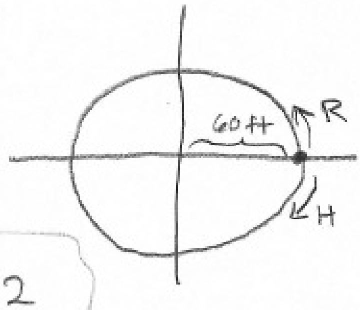
screen height = 50.06 feet ← a + 2

4. (10 points) Harry and Ron are both at the easternmost point of a circular track. The track has radius 60 feet. Ron runs in the counterclockwise direction at an angular speed of 0.02 radians per second. Harry runs in the clockwise direction.

(a) Impose a coordinate system with the middle of the track as the origin. Find the x and y coordinates of Ron after 30 seconds.

$x = r\cos(\omega t + \theta_0)$

$y = r\sin(\omega t + \theta_0)$



+5

$$\boxed{\begin{aligned} x &= 60\cos(0.02(30) + 0) \approx 49.52 \\ y &= 60\sin(0.02(30) + 0) \approx 33.88 \end{aligned}}$$

(b) How fast (in feet per second) must Harry run in order to be at the same location as Ron after 30 seconds? (Hint: You may want to consider the angle that Harry will travel.)

$\theta = \omega t$

Ron travels $\theta = 0.02 \times 30 = 0.6$ radians

Harry travels $2\pi - 0.6 \approx 5.68318531$ radians (in the negative direction) } +3

For Harry $\rightarrow$ $\omega = \dfrac{-5.68318531 \text{ radians}}{30 \text{ sec}} = 0.189439510239 \ \dfrac{\text{rad}}{\text{sec}}$ } +1

$v = \omega r = 0.189439510239 \times 60$

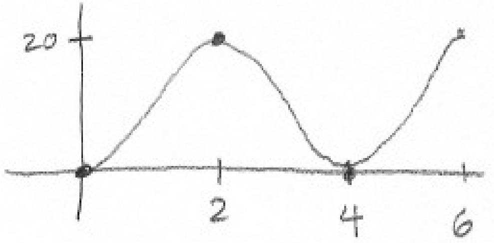
$= 11.3663706144$ feet/sec } +1

$$\boxed{v = 11.37 \text{ ft/sec}}$$

5. (10 points) Mr. Pogo can bounce on his giant pogo stick for hours at a time. When he is bouncing, Mr. Pogo's height off the ground changes according to a sinusoidal function of time. At time $t = 0$ seconds, Mr. Pogo is at his lowest height of 0 inches. At time $t = 2$ seconds, Mr. Pogo reaches his highest point of 20 inches for the first time.

(a) Find the sinusoidal model, $h(t)$, which gives height in terms of time.

+2 $h(t) = A\sin\left[\frac{2\pi}{B}(t-C)\right] + D$



+2 $A = \frac{20-0}{2} = 10$

+2 $D = \frac{20+0}{2} = 10$

+2 $B = 4$

+2 $C = 2 - \frac{4}{4} = 1$ ← not unique (could also use $1 \pm 4K$, K an integer)

$$\boxed{h(t) = 10\sin\left[\frac{2\pi}{4}(t-1)\right] + 10}$$

$= 10 \sin [$

(b) Find the height of Mr. Pogo after $t = 11$ seconds.

+2 $h(11) = 10\sin\left[\frac{2\pi}{4}(11-1)\right] + 10 = 10$ inches

or use your picture