1. (8 points) Let $f(x) = 1 + 2x - 3x^2$.

(a) Simplify the following expression far enough so that plugging in $h = 0$ would be allowed:

$$\frac{f(x+h) - f(x)}{h}$$

$$\frac{[1 + 2(x+h) - 3(x+h)^2] - [1 + 2x - 3x^2]}{h}$$

$$= \frac{\cancel{1} + \cancel{2x} + 2h - \cancel{3x^2} - 6xh - 3h^2 - \cancel{1} - \cancel{2x} + \cancel{3x^2}}{h}$$

$$= \frac{2h - 6xh - 3h^2}{h} = \boxed{2 - 6x - 3h}$$

(b) Simplify the following expression far enough so that plugging in $a = 0$ would be allowed:

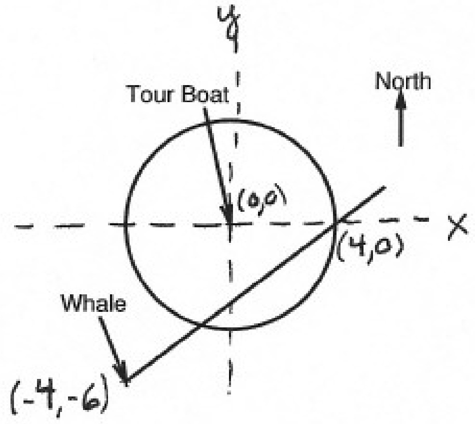
$$\frac{f(2a) - f(a)}{a}$$

$$\frac{[1 + 4a - 12a^2] - [1 + 2a - 3a^2]}{a}$$

$$= \frac{\cancel{1} + 4a - 12a^2 - \cancel{1} - 2a + 3a^2}{a}$$

$$= \frac{2a - 9a^2}{a} = \boxed{2 - 9a}$$

2. (10 points) A tour boat for whale watchers is sitting in one spot out on the ocean. The boat has a radar that will detect any whale within a radius of 4 miles. A whale is currently located 4 miles west and 6 miles south of the boat. The whale travels directly toward the easternmost edge of the radar zone at 10 mph. How long (in hours) will the whale be in the radar zone?



Circle: $\boxed{x^2+y^2=4^2}$

Line: $m = \frac{0 - -6}{4 - -4} = \frac{6}{8} = 0.75$

$$\underbrace{\boxed{y = 0.75(x-4)+0}}_{\text{ALSO CAN WRITE AS}}$$

$y = 0.75(x+4) \cancel{+} -6$ and

$y = 0.75x - 3$

Note: Other origins may be used. This will effect the equations of the circle and line, but the technique and final answer are still the same.

Intersection

$$x^2 + (0.75x-3)^2 = 4^2$$

$$x^2 + 0.5625x^2 - 4.5x + 9 = 16$$

$$1.5625x^2 - 4.5x - 7 = 0$$

$$x = \frac{4.5 \pm \sqrt{(-4.5)^2 - 4(1.5625)(-7)}}{2(1.5625)} = \frac{4.5 \pm 8}{3.125}$$

$x = -1.12$ or $x = 4$

$y = 0.75(-1.12) - 3 = -3.84$ $\qquad y = 0$

Time

$$\text{Time} = \frac{\text{Dist}}{\text{Speed}} = \frac{\sqrt{(4 - -1.12)^2 + (0 - -3.84)^2}}{10}$$

$$= \frac{\sqrt{40.96}}{10} = \frac{6.4}{10} = \boxed{0.64 \text{ hours}}$$

3. (10 points) Kramer has an apple orchard. One season, his 200 trees yielded 100 apples per tree, which gives a production of 20,000 apples. Kramer has a limited amount of space and if he adds more trees to his orchard it will decrease his yield per tree. For every 10 additional trees that Kramer adds to his orchard, he will lose 6 apples per tree.

(a) Find a linear model which gives the number of apples per tree, y, in terms of the number of trees, x.

$(200, 100) \qquad (210, 94)$

$$m = \frac{100-94}{200-210} = \frac{6}{-10} = -0.6$$

$$\boxed{y = -0.6(x-200)+100}$$

ALSO CORRECT: $y = -0.6(x-210)+94$

$y = -0.6x + 220$

(b) Write a formula for the total production of apples, $P(x)$, in terms of trees, x, and use this formula to determine how many trees Kramer should have in the orchard to maximize his production of apples. (Round your final answer to the nearest tree.)

$P(x) = x$ (function from part (a))

$= x[-0.6x + 220]$

$= -0.6x^2 + 220x$

For this parabola, the maximum is at the vertex:

$$h = -\frac{b}{2a} = -\frac{220}{2(-0.6)} = 183.\overline{3}$$

Rounding to the nearest tree gives

$\boxed{183 \text{ trees}}$

4. (12 points) Let $g(x) = x + 7$, $h(x) = x^2 + 1$, and $f(x) = \begin{cases} x^2 + 2 & , \text{ if } x \geq 2 \\ 2x - 1 & , \text{ if } x < 2 \end{cases}$.

(a) The function $h(g(x))$ is quadratic.
Give the formula for $h(g(x))$ and find the x and y coordinates of the vertex.

$$h(g(x)) = h(x+7) = (x+7)^2 + 1$$

$$\boxed{h(g(x)) = x^2 + 14x + 50}$$

Vertex:

$$x = -\frac{b}{2a} = -\frac{14}{2} = \boxed{-7}$$

$$y = (-7)^2 + 14(-7) + 50 = \boxed{1}$$

(b) Find all solutions for x in the following equation:

$$f(x) = 11$$

$x^2 + 2 = 11$, $x \geq 2$ or $2x - 1 = 11$ $x < 2$

$x^2 = 9$ $\qquad$ $2x = 12$

$\boxed{x = +3}$ or $\cancel{x = -3}$ in domain, not in domain $\qquad$ $\cancel{x = 6}$ not in domain

$$\boxed{x = 3 \text{ is the only sol'n}}$$

(c) Write the multipart rule for $f(g(x))$.

$$\begin{cases} (x+7)^2 + 2 & , \text{ if } x + 7 \geq 2 \\ 2(x+7) - 1 & , \text{ if } x + 7 < 2 \end{cases}$$

$$\boxed{= \begin{cases} (x+7)^2 + 2 & , \text{ if } x \geq -5 \\ 2x + 13 & , \text{ if } x < -5 \end{cases}}$$