Math 120 A - Winter 2006
Mid-Term Exam Number One
Answers
January 26, 2006

There were two versions. In version A, the forest in problem 1 had a radius of 6 km. In version B, the radius was 7 km.

1. To solve this, impose the coordinate system with the origin at the center of the forest and determine the coordinates of the starting point and the westernmost point of the forest. Find the equation of the line through those points. She will change direction at the y-intercept of that line, and will stay on the horizontal line through that point until leaving the forest. Find the intersection (in the third quadrant) of that horizontal line and the circle of the forest to get the final answer:

   Version A: (-3.9686,-4.5)

   Version B: (-4.9990,-4.9)

2. There were many ways to get to the solution for this problem. For $x < 6$, a vertical line will cut off a trapezoidal region which can be viewed as a triangle on top of a rectangle. Add their areas to get the value of the function at x. For $x > 6$, the area will be the area of the trapezoid of $x = 6$ plus the rectangular region to the right of 6; this rectangle has area $3(x - 6)$.

   After simplifying your answer, the function is:

   Version A:
   $$A(x) = \begin{cases} -\frac{2}{3}x^2 + 11x & \text{if } x < 6 \\ 3x + 24 & \text{if } x \leq 6 \end{cases}$$

   Version B:
   $$A(x) = \begin{cases} -\frac{7}{12}x^2 + 10x & \text{if } x < 6 \\ 3x + 21 & \text{if } x \leq 6 \end{cases}$$

   Note: the value of the function at $x = 6$ could be defined in either part of the function. In other words, you could have $x \leq 6$ and $x > 6$ as the two parts' domains.

3. You want first to find the linear function giving Ribbit's population, $R(t)$, in terms of time. Then use that function to find the population of Croak in 1990 and 1995, from which you can determine the linear function giving Croak's population, $C(t)$ in terms of time. Then solve the equation
   $$C(t) = 10000 + R(t) \qquad \text{(version A)}$$
   or
   $$C(t) = 5000 + R(t) \qquad \text{(version B)}$$
   to find the answer:

Version A: $t = 2011.5$

Version B: $t = 2009.3571$

4. Let $T(x)$ be the amount of money she makes at a ticket price of x. Then $T(x) = ax^2 + bx + c$ for some a, b, c. Using the three data points given, fina a, b, and c. Find the vertex of this quadratic. Since $a < 0$, the vertex represents the maximum value of this function: its x-coordinate is the price that yields the most money for her (the answer to part (a)) and the y-coordinate of the vertex is the most money she could make (the answer to part (b)).

   Version A: (a) $5.5 (b) $302.5

   Version B: (a) $4.1666 (b) $260.4166