Math 120 Exam II - Version A - Winter 2005 - Schneider

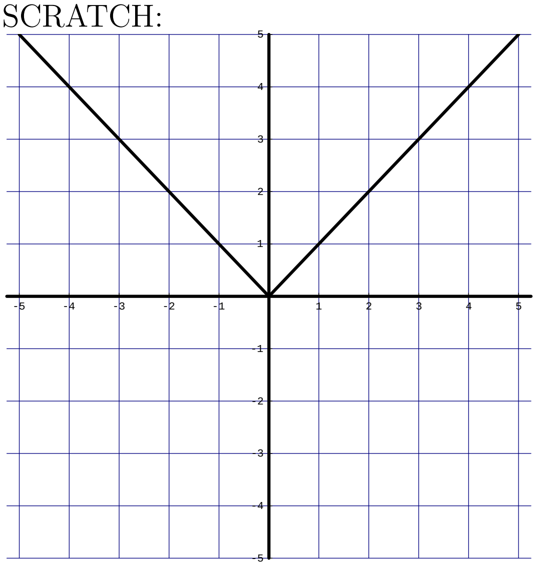
Problem 1. Let $g(x) = -1.5 \mid 2(x-2) \mid +3$.

[4 pts](a) Write down the precise order of graphical operations involved in going from the graph of $y = \mid x \mid$ to $g(x)$ and number them from first to last. With a shift or dilation, give the amount by which one shifts or dilates (remember, a shift to the right is positive, to the left is negative).

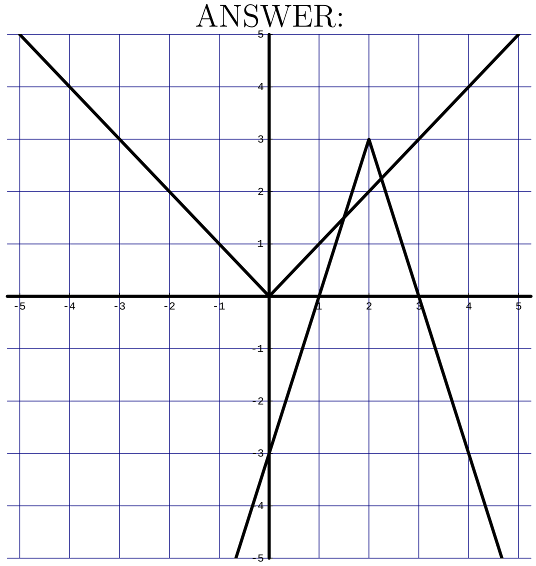
1. Horizontal Dilation by $\frac{1}{2}$
2. Horizontal shift to the right by 2
3. Vertical Dilation by 1.5
4. Reflection around x axis
5. Vertical Shift by 3.

[8 pts](b) Two copies of the graph of $\mid x \mid$ are given below. The first may be used for intermediate steps. In the second, sketch the graph of $g(x)$. The graph of $g(x)$ should have the vertex in the correct spot. If $g(x)$ intersects the x or y axis, your graph should intersect the axis at the correct points (you will give those points below).

SCRATCH:



ANSWER:



Since many students have calculators that will graph this function, little to no credit was given for just graphing the correct answer without seeing some scratch work.

What are the coordinates of the "vertex" of $g(x)$:

$(2, 3)$ (we can read this from the graph.)

If the graph of $g(x)$ intersects the x or y axis, what are the coordinates of these points of intersection?

$(0, -3), (1, 0), (3, 0)$

Problem 2. Dr. Bolzman has determined that during his chemical reaction, the amount of substance A is given by

$$A(t) = .18\,t^2 - 2\,t + 13.5$$

where t is in seconds and $A(t)$ is in grams. The reaction takes place between time $t = 0$ and $t = 10$. During these 10 seconds, there is a time when the amount of substance A reaches a minimum and then increases again.

[8 pts](a) Restrict $A(t)$ to the domain where the amount of substance A is increasing and find $A^{-1}(x)$.

The vertex for $A(t)$ is $(5.556, 7.9444)$ so we are restricting $A(t)$ to $[5.556, 10]$. One writes $A(t)$ in vertex form:

$$A(t) = .18(t - 5.556)^2 + 7.944$$

and solves for t to get (taking a positive square root since we are inverting to the right of the vertex):

$$t = \sqrt{\frac{A - 7.944}{.18}} + 5.556$$

So then

$$A^{-1}(x) = \sqrt{\frac{x - 7.944}{.18}} + 5.556$$

[2 pts](b) What is the range of $A^{-1}(x)$?

This is the old domain, so it is $[5.556, 10]$.

[2 pts](c) What is the domain of $A^{-1}(x)$?

This will be the range of $A(t)$, so we need to know what $A(10)$ is. $A(10) = 11.5$ so the answer is $[7.944, 11.5]$

Problem 3. Find the linear to linear function $f(x)$ such that

$$f(1) = 2 \qquad f(2) = 6 \qquad f(4) = 7$$

Since $f(x)$ is linear to linear, it looks like $\frac{ax+b}{x+d}$. First one write the three equations that the three points determine:

$$\frac{a+b}{1+d} = 2 \qquad \frac{2a+b}{2+d} = 6 \qquad \frac{4a+b}{4+d} = 7$$

Next one can solve for b in the first equation:

$$b = 2 + 2d - a$$

Then one can eliminate b from the next two equations. After a little algebra, one gets

$$2 + a + 2d = 12 + 6d \qquad 2 + 3a + 2d = 28 + 7d$$

From the first of these two equations

$$a = 10 + 4d$$

putting this in the second equation gives

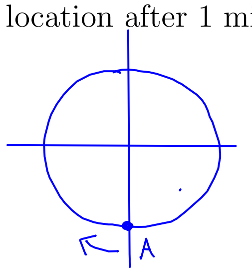
$$32 + 14d = 28 + 7d$$

or $d = -\frac{4}{7}$. one then goes back and gets $a = -\frac{54}{7}$ and $b = -\frac{48}{7}$. Then

$$f(x) = \frac{-\frac{54}{7}x - \frac{48}{7}}{x - \frac{4}{7}} = \frac{-54x - 48}{7x - 4}$$

Problem 4.

[6 pts](a) Alice is running 7 miles per hour clockwise around a circular track of radius 300 feet. Take the origin of a coordinate system to be at the center of the circular track. Assume Alice starts on the negative y axis as in the figure. What are the coordinates of Alice's location after 1 minute and 20 seconds?



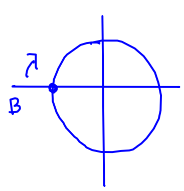
Alice starts at $\frac{3\pi}{2}$ and goes counterclockwise, so we subtract $\theta = \omega t$ to get Alice's angle. As $v = 7$mph, $\omega = \frac{v}{r}$.

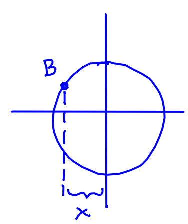
$$v = 7\frac{\text{miles}}{\text{hours}} = 36960\frac{\text{feet}}{\text{hours}} = 616\frac{\text{feet}}{\text{minute}}$$

Thus $\omega = 2.05333\frac{\text{rad}}{\text{minute}}$. Then $t = 1 + \frac{20}{60} = \frac{4}{3}$. So $\theta = 2.73778$ radians. Then Alice's angle is $\frac{3\pi}{2} - 2.73778 = 1.97461$.

One computes $x = 300\cos(1.97461) = -117.879$ and $y = 300\sin(1.97461) = 275.871$.

[6 pts](b) Bob makes four clockwise revolutions around a circular track every three minutes. Take the origin of a coordinate system to be at the center of the circular track. He starts on the negative x axis as in the figure. After 5 seconds, his x coordinate is -120.12 feet. Find the radius of the circular track.

$t = 0$: 

$t = 5$ seconds: 

Bob's $\omega = \frac{8\pi}{3}\frac{\text{rad}}{\text{min}}$. The angle he travels is ωt, or after 5 seconds, $\frac{8\pi}{3}\frac{5}{60} = \frac{2\pi}{9}$ radians. So his final angle is $\pi - \frac{2\pi}{9}$. Thus his x coord is $r\cos(\frac{7\pi}{9})$. We evaluate the cos and set this equal to the given x value:

$$r(-0.766044) = -120.12 \quad \Rightarrow r = 156.806.$$