Math 120AB - Winter 2004
Mid-Term Exam Number Two Answers/Solutions
February 26, 2004

1. Let $f(x) = \sqrt{x} + \sqrt{x+2}$. Find $f^{-1}(x)$.

   Solution:

   Let
$$y = \sqrt{x} + \sqrt{x+2}$$
   and solve for x:
$$\begin{aligned} y - \sqrt{x} &= \sqrt{x+2} \\ (y-\sqrt{x})^2 &= x+2 \\ y^2 - 2y\sqrt{x} + x &= x+2 \\ y^2 - 2y\sqrt{x} &= 2 \\ -2y\sqrt{x} &= 2 - y^2 \\ \sqrt{x} &= \frac{y^2-2}{2y} \\ x &= \left(\frac{y^2-2}{2y}\right)^2. \end{aligned}$$
   So
$$f^{-1}(x) = \left(\frac{x^2-2}{2x}\right)^2$$

2. Sue has a tree in her yard which has a height determined by a linear-to-linear rational function of time. When the tree was planted, it had a height of zero feet (it was just a seed). Ten years later, it had a height of 30 feet. Twenty years after it was planted, its height was 55 feet. In the long run, how tall will the tree be?

   Solution:

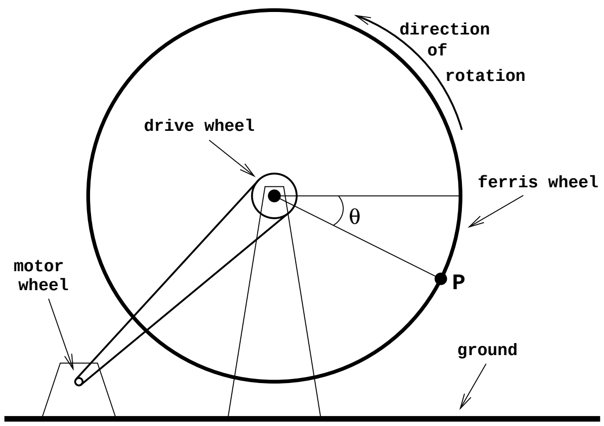
   Let $h(t)$ be the height of the tree t years after planting. Then
$$h(t) = \frac{at+b}{t+c}$$

for constants a, b, c. Also, $h(0) = 0$, $h(10) = 30$, and $h(20) = 55$. So $b = 0$, $a = 330$, and $c = 100$. That is,

$$h(t) = \frac{330t}{100 + t}.$$

This function has a horizontal asymptote of $y = 330$, so in the long run, we can say the tree will be 330 feet tall.

3. Paul is riding a ferris wheel with a diameter of 250 feet. It is powered by a 1.2 foot diameter motor wheel, which is attached by a chain to a 26 foot diameter drive wheel, as shown in the diagram. The drive wheel is attached to the same axle as the ferris wheel. The axle of the ferris wheel is 140 feet above the ground.



(a) How fast is the motor wheel turning if Paul is moving 25 feet per second?

Solution:

The angular speed of the ferris wheel is

$$\frac{25}{125} \text{ radians/sec.}$$

which is also the angular speed of the drive wheel. The linear speed of the drive wheel is

$$\frac{25}{125}13$$

feet per second. This is also the linear speed of the motor wheel, so the angular speed of the motor wheel is

$$\frac{\frac{25}{125}13}{0.6} = 4.333333...$$

radians per second.

(b) Suppose Paul is moving 25 feet per second and the wheel is rotating counter-clockwise. If $\theta = 23^\circ$, how high above the ground is Paul 3 seconds after being at point P?

Solution:

The angular speed of Paul is $\omega = \frac{25}{125}$ radians per second. Thus t seconds after being at point P Paul will be

$$h(t) = 140 + 125\sin(\omega t - 23^\circ)$$

feet off the ground. Setting $t = 3$ and being careful with units we find

$$h(3) = 164.658977$$

feet.

4. At a certain point in the desert, the temperature is given by a sinusoidal function of time. The high and low daily temperatures occur exactly once each per day. The low daily temperature is 35° and it occurs at 3 AM. The high daily temperature is 104° and it occurs at 3 PM.

   What is the temperature at noon?

   Solution:

   Letting t represent the number of hours since midnight, from the information given we can find

$$A = (104 - 35)/2 = 34.5$$

$$D = (104 + 35)/2 = 69.5$$

$$B = (2)(12) = 24$$

$$C = 3 + 24/4 = 9$$

So the temperature is given by

$$T(t) = 69.5 + 34.5\sin(\frac{2\pi}{24}(t-9))$$

and the temperature at noon is

$$T(12) = 93.8951839^\circ.$$

5. You are watching a rocket launch. A short time after take-off, the rocket appears to be 68° high (i.e., a line from you to the rocket makes a 68° angle with the horizontal). A little later, the rocket has climbed an additional 100 meters, and now appears to be 70° high. How far are you from the launch pad?

   Solution:

   Using trigonometry (specifically the tangent relationship between sides of a right triangle), we find the distance to the launch pad to be

$$\frac{100}{\tan 70^\circ - \tan 68^\circ} = 367.119909673852 \text{ meters.}$$