1. (5 points) A point is moving in the $xy$-coordinate system according to the parametric equations:

$$x(t) = 100 \sin\left(\frac{2\pi}{7}(t - 6)\right) + 200$$

$$y(t) = 100 \sin\left(\frac{2\pi}{7}(t - 0.5)\right) + 200$$

where $t$ represents time in seconds and $0 \leq t \leq 5$. Here is a graph of this parametrized curve.

(a) (3pts) Where is the object located at time $t = 0$ seconds? (Find the coordinates and mark in the picture.)

$P(0) = (278.2, 156.6)$; point should be labeled on the curve.

(b) (1pts) Which direction is the point moving along the curve? (Indicate with some sort of arrow.)

Moves from $P(0)$ in part (a) to the other end of the curve.

(c) (1 pts) Can you find a function $y = f(x)$ whose graph is this parametrized curve? (Justify)

No; curve violates the vertical line test.
2. Barney and Spencer start at the two locations pictured in the $xy$-coordinate system; the units on each axis will be meters. Barney is initially located at the point $(0,-8)$ and has a speed of 3 meters/sec in the direction indicated. Spencer has parametric equations: $S(t) = (x_S(t), y_S(t))$, where $x_S(t) = 1 + t, y_S(t) = 3 - 2t$. Barney and Spencer start moving at the same time.

(a) (2pts) Find Spencer’s initial coordinates at time $t = 0$.

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(b) (1pts) Find the time when Spencer crosses the $x$-axis.

Solve $y_S(t) = 0$; get $t = 3/2$, plug into $S(3/2) = (5/2, 0)$ to be the place Spencer crosses the $x$-axis.

(c) (5pts) Find parametric equations for Barney’s motion.

Resolve Barney’s velocity vector; the direction angle is $\frac{\pi}{2} - 0.3 = 1.271$ rad. Get $v_x = 0.886$ m/s and $v_y = 2.866$ m/s. Conclude Barney’s position at time $t$ is given by $B(t) = (0.886t, -8 + 2.866t)$.

(d) (4pts) Find the distance between Barney and Spencer at time $t$. Your answer should be a function in the variable $t$. Use your formula to calculate the distance between these guys at time $t = 2$ seconds.

Use the distance formula to get $d(t) = \sqrt{(0.886t - 1 - t)^2 + (-8 + 2.866t - 3 + 2t)^2}$. This answer is ok, could simplify to get $d(t) = \sqrt{23.691t^2 - 106.82t + 122}$. Calculate $d(2) = 1.77$ meters.

(e) (3pts) What is the velocity vector for Spencer? (You need to specify Spencer’s speed and direction.)

Speed is $|v| = \sqrt{(-2)^2 + 1} = 2.24$ m/s. Direction is $\theta = -1.107$ rad or $\theta = 5.176$ rad.