1. (18pt) The value of a house in Seattle in the year $t$ is modeled by the linear function

$$s(t) = 7000(t - 1970) + 40000 \text{ dollars}$$

on the domain $1970 \leq t$.

On the other hand, in La Jolla (California), the value of a house was $98,000 in 1970 and $202,000 in 1990. Assume the value of the house in the year $t$ in La Jolla is modeled by a linear function $j(t)$ on the domain $1970 \leq t$.

(a) (2pt) What was the value of a house in Seattle in 1983? (Round to nearest dollar.)

Plug 1983 into the model for the Seattle house value:

$$s(1983) = 7000(1983 - 1970) + 40000 = 131,000;$$

(b) (4pt) Here is a picture of the graph of $s(t)$. What are the coordinates of the two points labeled $A$ and $B$? (Round to one decimal place.)

Every point on the graph looks like $(t, s(t))$, so $A$ is a point where $s(t) = 50000$ is given to us and we need to find $t$. We have to solve this equation for $t$

$$s(t) = 50000 \text{ i.e., } 7000(t - 1970) + 40000 = 50000.$$

You will get 1971.4. So, $A = (1971.4, 50000)$

If you know the $t$ coordinate of a point on the graph, then the $y$ coordinate is the function evaluated at that $t$ coordinate. For this example,


[Graph of s(t) and j(t) showing points A and B]
(c) (5pt) Find the formula for \( j(t) \). Sketch the graph of \( j(t) \) in the coordinate system of part (b).

The problem gives us two data points on the line representing La Jolla house values: (1970,98000) and (1990,202000). Use the two point formula for a line:

\[
y = \frac{202000 - 98000}{1990 - 1970} (t - 1970) + 98000
\]

\[
= 5200(t - 1970) + 98000
\]

So, \( j(t) = 5200(t - 1970) + 98000 \). The graph is sketched in part (b) along with the two data points.

(d) (5pt) When will the value of a house in La Jolla exceed the value of a house in Seattle by exactly $5,000? (Carry out to one decimal place.)

We need to write down the equation to solve; this amounts to translating the words into an equation:

\[
\text{value house in La Jolla exceed the value of house Seattle by exactly } 5000
\]

\[
\begin{align*}
\quad j(t) &= s(t) + 5000 \\
5200(t - 1970) + 98000 &= 7000(t - 1970) + 40000 + 5000 \\
5200t - 1014600 &= 7000t - 13745000 \\
3599000 &= 1800t \\
1999.4 &= t
\end{align*}
\]

The answer is the year 1999.4 (rounded to one decimal place.)

(e) (2pt) Consider the function \( s(t) \) on the NEW domain \( 1990 \leq t \leq 2020 \). What is the range of this function?

The graph of this function is a piece of the line graphed in (b) for the graph of \( s(t) \). You need to consider only the part of the line which is above the domain on the \( t \)-axis. That means the part of the graph above the interval \( 1990 \leq t \leq 2020 \). On this interval, you can see from the picture that \( s(1990) = 180000 \) is the smallest value and \( s(2020) = 390000 \) is the largest value. Conclude the range is \( 180000 \leq y \leq 390000 \).

2. (2pt) Consider the function

\[
f(x) = \frac{-x}{x^2 + 1}
\]

on the domain of all real numbers. Compute

(a) \( f(-3) = \frac{-(-3)}{(-3)^2 + 1} = \frac{3}{10} = 0.3 \).

(b) \( f(1 - 2x) = \frac{-(1-2x)}{(1-2x)^2 + 1} = \frac{-2x-1}{(1-2x)^2 + 1} = \frac{2x-1}{4x^2 - 4x + 2} \).