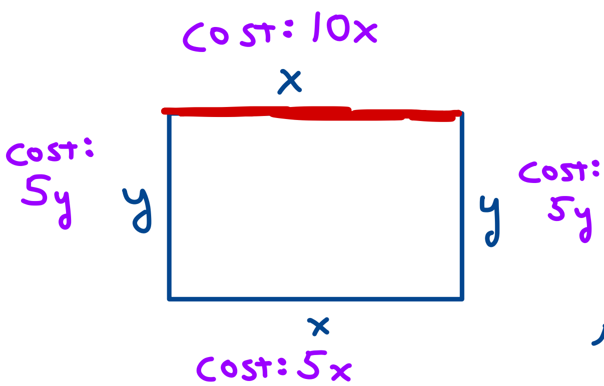
1. **[15 points]** I have $1200 to build a rectangular enclosure.

Three of the sides use regular fencing that costs $5 per foot, but one of the sides uses super fencing[1] that costs $10 per foot.

What is the maximum possible area of this enclosure?



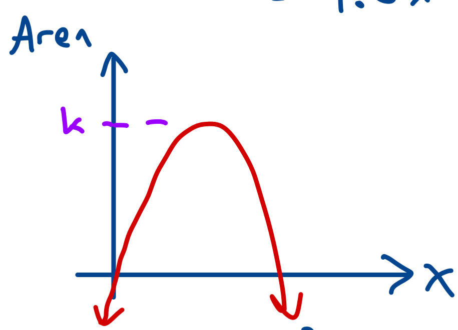
$$\text{Area} = xy$$

$$1200 = 15x + 10y$$

$$y = 120 - 1.5x$$

$$\text{Area} = x(120 - 1.5x)$$

$$= -1.5x^2 + 120x$$



$$k = 0 - \frac{120^2}{4(-1.5)}$$

Area: $\boxed{2400}$ square feet

[1] Look, I know "super fencing" isn't a thing, I'm just running out of ideas here.

2. Suppose f is a linear-to-linear rational function with the following properties:

- $f(3) = -4$
- $f(25) = 8$
- The graph of f has a horizontal asympote of $y = 5$.

(a) **[12 points]** Find a formula for $f(x)$.

$$f(x) = \frac{ax+b}{x+d}$$

$$a = 5$$

$$-4 = \frac{3a+b}{3+d} \rightarrow -12-4d = 3a+b \rightarrow -12-4d = 15+b \rightarrow b = -27-4d$$

$$8 = \frac{25a+b}{25+d} \rightarrow 200+8d = 25a+b \rightarrow 200+8d = 125+b \rightarrow b = 75+8d$$

$$-27-4d = 75+8d$$

$$-12d = 102$$

$$d = -8.5$$

$$b = -27-4(-8.5) = 7$$

$f(x) = \boxed{\dfrac{5x+7}{x-8.5}}$

(b) **[3 points]** What is the domain of f?

Everything but the vertical asymptote

$x = -d = 8.5$

Domain: $\boxed{(-\infty, 8.5) \cup (8.5, \infty)}$

3. **[7 points per part]** Carmy's restaurant is growing exponentially in popularity.

In the year 2020, there were 10 thousand customers.

In the year 2024, there were 13.6 thousand customers.

(a) Write a function $c(t)$ for the number of customers, in thousands, t years after 2020.

Write your answer in standard exponential form.

$c(t) = A_0 b^t$

$A_0 = 10$ (# when $t=0$)

$13.6 = 10 \cdot b^4$

$b^4 = 1.36$

$b = 1.36^{1/4}$

$c(t) = \boxed{10 \cdot (1.36^{1/4})^t}$

(b) When will there be 74 thousand customers? (Round to the nearest year.)

$74 = 10(1.36^{1/4})^t$

$7.4 = (1.36^{1/4})^t$

$t = \log_{(1.36^{1/4})} 7.4 = \dfrac{\log(7.4)}{\frac{1}{4}\log(1.36)} \approx \underbrace{26.04}_{\text{years after 2020}}$

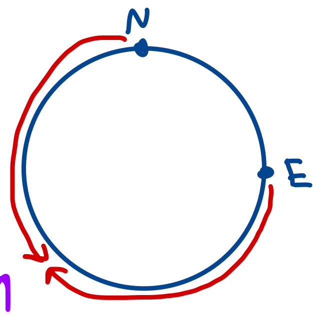
Year: $\boxed{2046}$

4. **[8 points]** Norris and Esau are running around a circular track with radius 15 m.

Norris begins at the northernmost point and runs counterclockwise at a speed of 6 m/s.

Esau begins at the easternmost point and runs clockwise at constant speed. It takes him 20 seconds to run a full lap.

When do they pass each other?



Total dist.
$\frac{3}{4}(2\pi \cdot 15) = 22.5\pi$

Norris: $v = 6$ m/s

Esau: $\omega = \frac{2\pi}{20} = \frac{\pi}{10}$ rad/s

$v = \omega r = \frac{15\pi}{10} = \frac{3\pi}{2}$ m/s

At time t, they've gone a total of $6t + \frac{3\pi}{2}t$ m.

When does this equal $\frac{3}{4}$ of the circle?

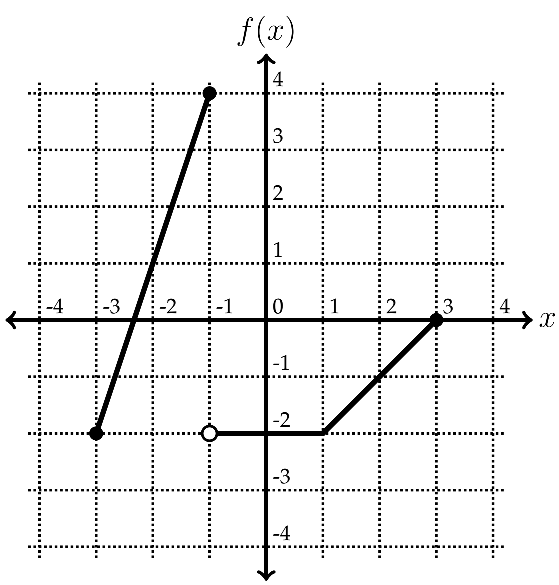
$$22.5\pi = \left(6 + \frac{3\pi}{2}\right)t$$

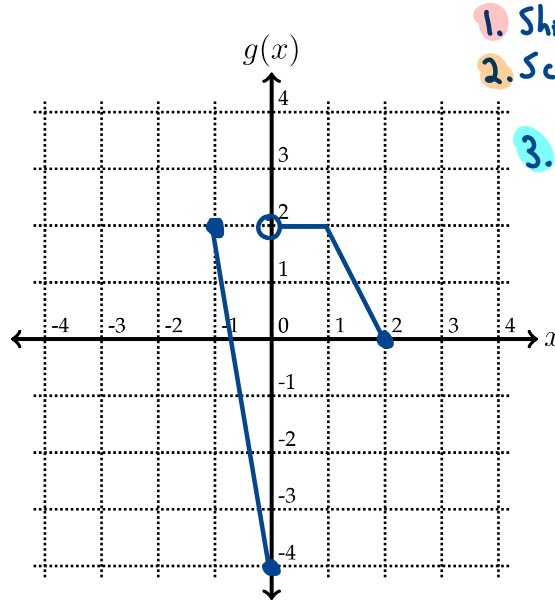
$$t = \frac{22.5\pi}{6 + \frac{3\pi}{2}}$$

After $\boxed{6.599}$ seconds

5. **[8 points]** On the left is the graph of $f(x)$. On the right, please graph $g(x) = -f(2x - 1)$.

1. Shift right 1
2. Scale horiz. by $\frac{1}{2}$
3. Reflect vertically





If you need extra space, there are some spare grids on the back of the exam.