1. **[15 points]** Vision Transit is building a new light rail track, which is a straight line.

   Part of the track has to cross a circular lake with radius 13 km.

   One end of the track, at Kudjins Park Station, is at the westernmost point of the lake.

   The other end, in Vellebue, is 17 km east and 20 km north of the center of the lake.

   It costs \$5 M per km to build track along the lake, and \$2 M per km to build track on land.

   How much does it cost to build the whole track?

$$\text{slope} = \frac{20-0}{17+13} = \frac{2}{3}$$

$$V \quad (17,20)$$

$$y = \frac{2}{3}(x+13)$$

$$(5,12)$$

$$(-13,0)$$

$$K$$
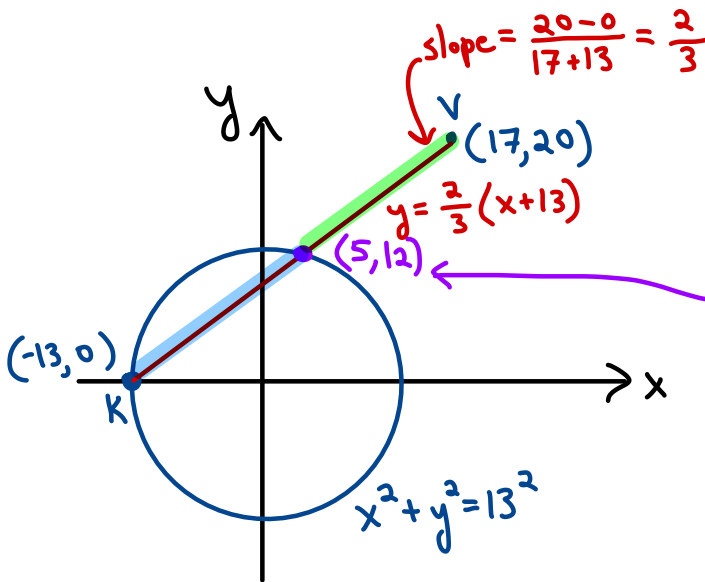
$$x^2 + y^2 = 13^2$$

Intersection:

$$y = \frac{2}{3}(x+13)$$

$$x^2 + y^2 = 13^2$$

$$x^2 + \left(\frac{2}{3}(x+13)\right)^2 = 13^2$$

$$x^2 + \frac{4}{9}(x^2 + 26x + 169) = 169$$

$$9x^2 + 4x^2 + 104x + 676 = 1521$$

$$13x^2 + 104x - 845 = 0$$

$$x^2 + 8x - 65 = 0$$

$$(x-5)(x+13) = 0$$

$$x = 5 \text{ or } -13$$

$$y = \frac{2}{3}(x+13) = 12$$

$$\text{Cost} = 5(\text{lake dist}) + 2(\text{land dist})$$

$$= 5\sqrt{18^2 + 12^2} + 2\sqrt{12^2 + 8^2}$$

$$= 30\sqrt{13} + 8\sqrt{13}$$

$$= 38\sqrt{13} \approx 137.01$$

Total cost: \$ __137.01__ M

2. Robin and Victoire are walking around the coordinate plane.

(a) **[6 points]** Robin starts at the point $(7, 2)$, and walks towards the point $(-3, -3)$ in a straight line at a constant speed, reaching it after 8 seconds.

Write parametric equations for Robin's location after t seconds.

$$x_0 = 7 \quad y_0 = 2$$
$$x_1 = -3 \quad y_1 = -3$$
$$\Delta x = -10 \quad \Delta y = -5$$
$$\Delta t = 8$$

$$x = 7 - \frac{10}{8}t$$
$$y = 2 - \frac{5}{8}t$$

$$\boxed{\begin{array}{l} x = 7 - \frac{5}{4}t \\ y = 2 - \frac{5}{8}t \end{array}}$$

Parametric equations: _____

(b) **[9 points]** Victoire starts at the point $(-5, 6)$ and walks in a straight line towards the point where Robin crosses the x-axis, at a constant speed of 2 units per second.

Write parametric equations for Victoire's location after t seconds.

Where? $\quad y = 2 - \frac{5}{8}t = 0 \qquad x = 7 - \frac{5}{4}(3.2) = 3$

$$2 = \frac{5}{8}t \qquad \text{Robin crosses } x\text{-axis @ } (3, 0)$$

$$t = 3.2$$

Victoire:
$$x_0 = -5 \quad y_0 = 6$$
$$x_1 = 3 \quad y_1 = 0$$
$$\Delta x = 8 \quad \Delta y = -6$$
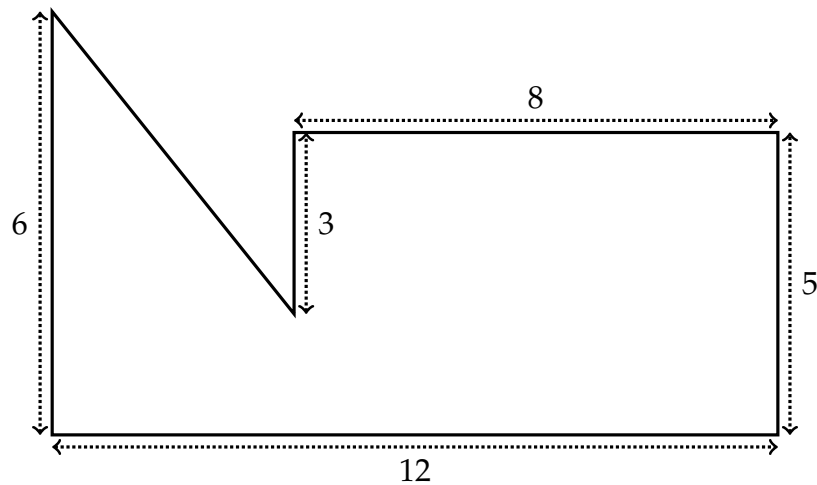
dist from $(-5, 6)$ to $(3, 0)$: $\sqrt{(-5-3)^2 + 6^2} = 10$

$$\Delta t = \frac{dist}{speed} = \frac{10}{2} = 5$$
(2)

$$\boxed{\begin{array}{l} x = -5 + \frac{8}{5}t \\ y = 6 - \frac{6}{5}t \end{array}}$$

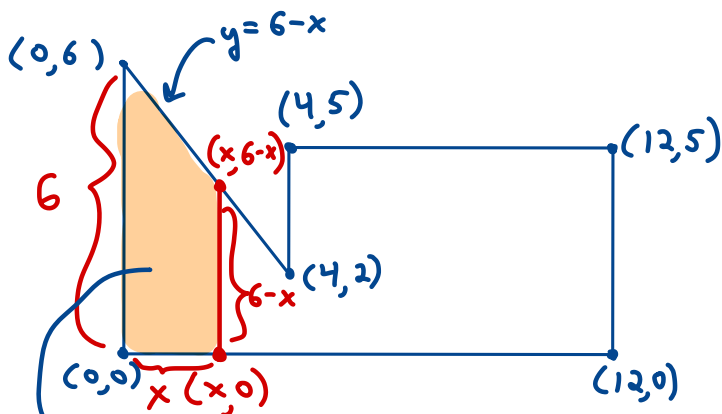Parametric equations: _____

3. **[15 points]** Precalculus Park is shaped like a weird hexagon, like this:

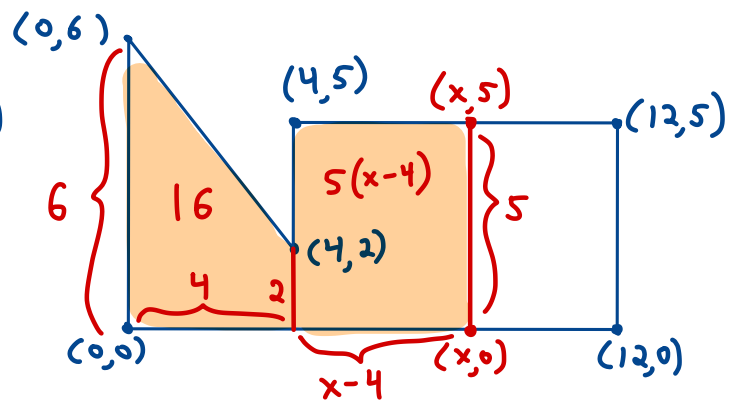Suppose a vertical line is drawn x units from the left edge of the park.

Write a multipart function $A(x)$ for the area to the left of the line.

If $0 \leq x \leq 4$:

$\text{Area} = \frac{1}{2}(6 + 6 - x)x$

$= 6x - \frac{1}{2}x^2$

If $4 \leq x \leq 12$

$\text{Area} = 16 + 5(x - 4)$

$= 5x - 4$

$A(x) = \begin{cases} 6x - \frac{1}{2}x^2 & \text{if } 0 \leq x \leq 4 \\ 5x - 4 & \text{if } 4 \leq x \leq 12 \end{cases}$

4. Suppose f is a quadratic function with the following properties:

- $f(2) = -5$
- $f(5) = 16$
- The vertex of the parabola $y = f(x)$ has an x-coordinate of 4.

(a) **[12 points]** Write a formula for $f(x)$.

$\rightarrow$ Easier in vertex form: we know $\boxed{h = 4}$

$f(x) = a(x-h)^2 + k$

$\rightarrow -5 = a(2-4)^2 + k \rightarrow -5 = 4a + k$

$\rightarrow 16 = a(5-4)^2 + k \rightarrow 16 = a + k$

$-21 = 3a$

$\boxed{a = -7}$

$\rightarrow 16 = -7 + k$

$\downarrow$

$\boxed{k = 23}$

or: $-7x^2 + 56x - 89$

$\uparrow$

$f(x) = \boxed{-7(x-4)^2 + 23}$

(b) **[3 points]** What is the range of f?

Parabola pointing <u>down</u> w/ vertex at $(4, 23)$,

So range is $f(x) \leq 23$

Range $\boxed{(-\infty, 23]}$