1. **[15 points]** Gomba is learning to climb a new set of stairs. The time it takes him to climb them on his n^{th} attempt is a linear-to-linear rational function of n.

   On his **1st** attempt, it took him 31 seconds to climb the stairs.

   On his **4th** attempt, it took him 17 seconds to climb the stairs.

   As Gomba continues to practice, the time it takes will approach (but not reach) 3 seconds.

   How long does his **13th** attempt take?

$$f(n) = \frac{an+b}{n+d}$$

$$f(1) = 31 \rightarrow \frac{a+b}{1+d} = 31 \rightarrow 3+b = 31+31d \rightarrow b = 28+31d$$

$$f(4) = 17 \rightarrow \frac{4a+b}{4+d} = 17 \rightarrow 12+b = 68+17d$$

y=3 is a horizontal asymptote $\rightarrow \boxed{a=3}$

$$12+28+31d = 68+17d$$

$$14d = 28$$

$$\boxed{d=2}$$

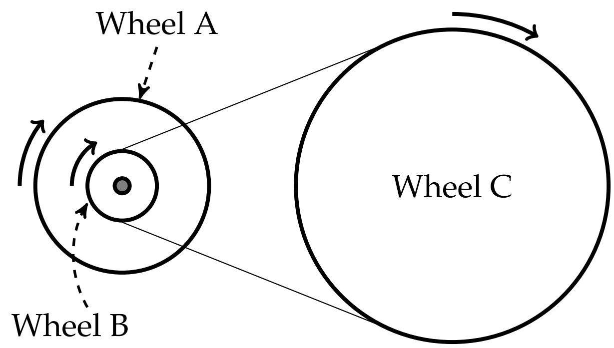
$$b = 28+31(2)$$

$$\boxed{b=90}$$

$$f(n) = \frac{3n+90}{n+2}$$

$$f(13) = \frac{3(13)+90}{13+2} = \boxed{8.6 \text{ seconds}}$$

2. **[15 points]** Three wheels are connected as shown in the diagram below: Wheels A and B are connected by an axle, and Wheels B and C are connected by a belt.



Wheel A has a radius of 5 meters, and rotates at a linear speed of 10 meters per second.

Wheel C has a radius of 8 meters, and takes $\underbrace{\text{9 seconds to make one complete rotation}}_{\text{angular speed is } \frac{2\pi}{9} \text{ rad/sec}}$.

What is the radius of wheel B?

Wheel	v (m/sec)	ω (rad/sec)	r (m)
A	10	2	5
B	$\frac{16\pi}{9}$	2	$\frac{8\pi}{9}$
C	$\frac{16\pi}{9}$	$\frac{2\pi}{9}$	8

$$\boxed{\frac{8\pi}{9} \text{ meters}}$$

3. **[5 points per part]** For each part of this question, let $f(x) = 3\log_2(x) + 2$.

(a) Find a formula for $f^{-1}(x)$. **Write your answer in standard exponential form.**

$3\log_2(x)+2=y$

$3\log_2(x)=y-2$

$\log_2(x)=\frac{y-2}{3}$

$x=2^{\frac{y-2}{3}}$

$f^{-1}(x)=2^{\frac{x-2}{3}}$

standard exp. form

$f^{-1}(x)=2^{\frac{x-2}{3}}=2^{\frac{-2}{3}}2^{\frac{x}{3}}$

$=\boxed{2^{\frac{-2}{3}}\left(2^{1/3}\right)^x}$

(b) Suppose $f(f(x)) = 11$. What's x?

$3\log_2(3\log_2(x)+2)+2=11$

$3\log_2(3\log_2(x)+2)=9$

$\log_2(3\log_2(x)+2)=3$

$3\log_2(x)+2=2^3=8$

$3\log_2(x)=6$

$\log_2(x)=2$

$x=2^2=\boxed{4}$

(c) Let $g(x) = \log_2(x)$. What transformations (shifting, scaling, reflecting) will lead you from the graph of $y = g(x)$ to the graph of $y = f(x)$?

$y=3\log_2(x)+2 \rightarrow \frac{y-2}{3}=\log_2(x)$

$y=\log_2(x) \xrightarrow{?} \frac{y-2}{3}=\log_2(x)$

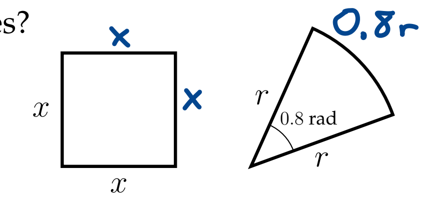
Fill in the blanks:

- First, you scale vertically by a factor of 3.
- Then, you shift 2 units up.

4. **[15 points]** Steve is no longer invited to parties, because he keeps trying to entertain people with optimization problems. Here's his most notorious trick:

He takes a 16 cm piece of wire, breaks it into two pieces, and uses those pieces to construct two figures: a square, and a sector with angle 0.8 radians.

What is the **minimum possible total area** of these two shapes?



$$A = \overbrace{x^2}^{\text{area of square}} + \overbrace{\tfrac{1}{2}(0.8)r^2}^{\text{area of sector}}$$

$$16 = \overbrace{4x}^{\text{perim of square}} + \overbrace{2.8r}^{\text{perim. of sector}}$$

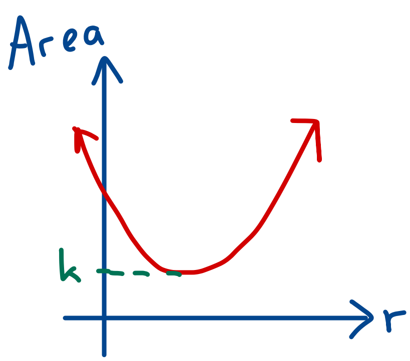
$$4x = 16 - 2.8r$$

$$x = 4 - 0.7r$$

$$A = (4-0.7r)^2 + 0.4r^2$$

$$= 16 - 5.6r + 0.49r^2 + 0.4r^2$$

$$= 0.89r^2 - 5.6r + 16$$



$$k = 16 - \frac{(-5.6)^2}{4(0.89)} \approx \boxed{7.191 \text{ square cm}}$$