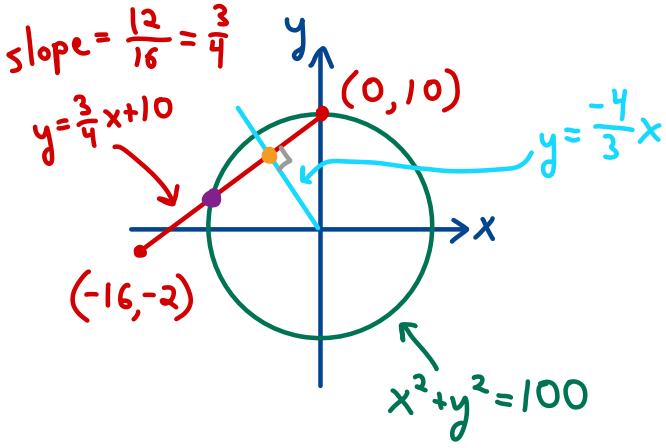
1. **[15 points]** Jim is standing at the center of a circular ring of radius 10 meters.

   At time $t = 0$, Naomi is 16 meters west and 2 meters south of Jim.

   Naomi runs in a straight line towards the northernmost point of the ring.

   Naomi runs at a constant speed of 5 meters per second while she's outside the ring, but slows down to 3 meters per second once she enters the ring.

   (a) When does Naomi enter the ring?



Where Naomi enters the ring

$y = \frac{3}{4}x + 10$ $\quad x^2 + y^2 = 10^2$

$x^2 + \left(\frac{3}{4}x + 10\right)^2 = 10^2$

$x^2 + \frac{9}{16}x^2 + 15x + 100 = 100$

$\frac{25}{16}x^2 + 15x = 0$

$x\left(\frac{25}{16}x + 15\right) = 0$

$x = 0$ or $x = \frac{-240}{25} = -9.6$, $y = \frac{3}{4}x + 10 = 2.8$

no

Dist from $(-16,-2)$ to $(-9.6, 2.8)$

$= \sqrt{(-16+9.6)^2 + (-2-2.8)^2} = 8$

time $= \frac{\text{dist}}{\text{speed}} = \frac{8}{5} =$ **1.6 sec**

   (b) When is Naomi closest to Jim?

Where Naomi is closest to Jim

$y = \frac{3}{4}x + 10$ $\quad y = \frac{-4}{3}x$

$\frac{3}{4}x + 10 = \frac{-4}{3}x$

$9x + 120 = -16x$

$25x = -120$

$x = -4.8$

$y = \frac{-4}{3}x = 6.4$

Dist from $(-9.6, 2.8)$ to $(-4.8, 6.4)$

$\sqrt{(-9.6+4.8)^2 + (2.8-6.4)^2} = 6$

total time $= \underbrace{\frac{8}{5}}_{\text{outside}} + \underbrace{\frac{6}{3}}_{\text{inside}} =$ **3.6 sec**

2. **[5 points per part]** Gregg and Bea are walking around the coordinate plane.

(a) Gregg starts at the point $(5, -2)$, and walks towards the point $(-13, 7)$ in a straight line at a constant speed, reaching it after 12 seconds.

Write parametric equations for Gregg's location after t seconds.

$x_0 = 5$ $\quad y_0 = -2$

$x_1 = -13$ $\quad y_1 = 7$

$\Delta x = -18$ $\quad \Delta y = 9$

$\Delta t = 12$

$x = 5 - \frac{18}{12}t \rightarrow x = 5 - \frac{3}{2}t$

$y = -2 + \frac{9}{12}t \rightarrow y = -2 + \frac{3}{4}t$

(b) Bea starts at the point $(-3, 4)$, and walks towards the point $(3, -4)$ at a constant speed of 4 units per second.

Write parametric equations for Bea's location after t seconds.

$x_0 = -3$ $\quad y_0 = 4$

$x_1 = 3$ $\quad y_1 = -4$

$\Delta x = 6$ $\quad \Delta y = 8$

$\text{distance} = \sqrt{6^2 + 8^2} = 10$

$\Delta t = \frac{\text{dist}}{\text{speed}} = \frac{10}{4} = 2.5$

$x = -3 + \frac{6}{2.5}t \rightarrow x = -3 + \frac{12}{5}t$

$y = 4 - \frac{8}{2.5}t \rightarrow y = 4 - \frac{16}{5}t$

(c) When is Gregg due north of Bea?

When their x-coordinates are equal.

$5 - \frac{3}{2}t = -3 + \frac{12}{5}t$

$5 + 3 = \frac{12}{5}t + \frac{3}{2}t$

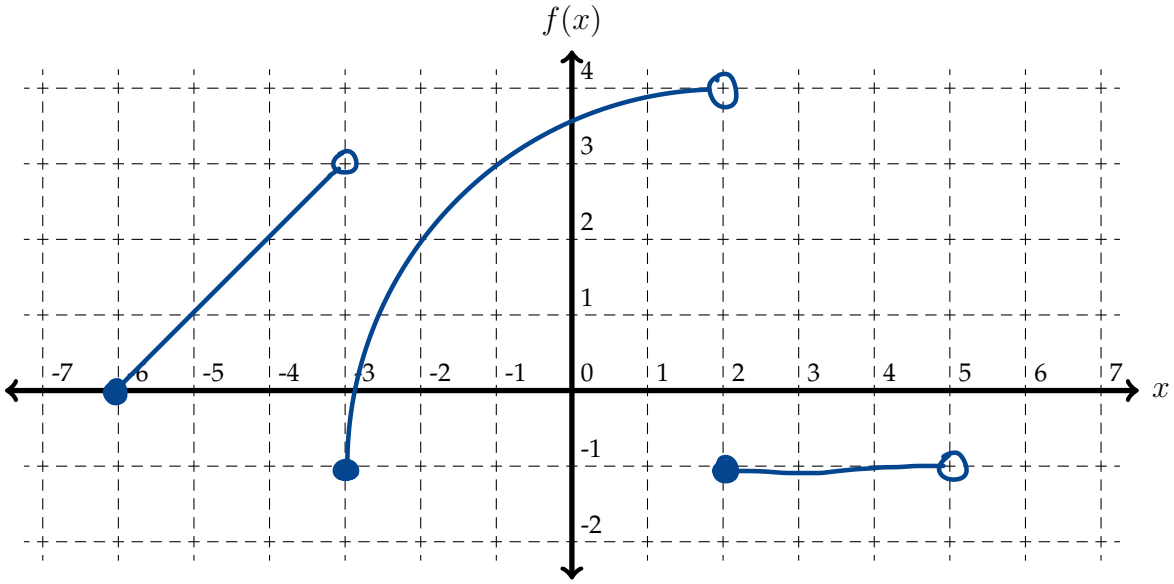
$8 = \frac{39}{10}t$

$t = \frac{80}{39}$ seconds ≈ 2.05128 seconds

3. For this problem, consider the following multipart function:

$$f(x) = \begin{cases} x+6 & \text{if } -6 \le x < -3 \\ -1 + \sqrt{25-(x-2)^2} & \text{if } -3 \le x < 2 \\ -1 & \text{if } 2 \le x < 5 \end{cases}$$

(a) **[6 points]** Sketch a graph of f here:



(b) **[3 points]** What is the range of f?

$[-1, 4)$

(the y-values in the above graph)

(c) **[6 points]** Find all values of x such that $f(x) = 2$.

$-6 \le x < 3$:

$x + 6 = 2$

$x = -4$

$-3 \le x < 2$:

$-1 + \sqrt{25-(x-2)^2} = 2$

$\sqrt{25-(x-2)^2} = 3$

$25-(x-2)^2 = 9$

$(x-2)^2 = 16$

$x - 2 = \pm 4$

$x = 2 \pm 4 = -2$ or ~~6~~ not in domain

$2 \le x < 5$:

$-1 = 2$

No solutions.

$x = -4$ or -2

4. Merlin is selling orbs. His profit is a quadratic function of how much he charges.

If Merlin gives away the orbs for free, he'll **lose** $200.

If Merlin charges $10 per orb, he'll **earn** a profit of $280.

If Merlin charges $20 per orb, he'll **earn** a profit of $700.

(a) **[12 points]** Write a function $f(x)$ for Merlin's profit if he charges $\$x$ per orb.

$f(x) = ax^2 + bx + c$

$f(0) = -200 \rightarrow -200 = c$

$f(10) = 280 \rightarrow 280 = 100a + 10b + c \rightarrow 10b = 480 - 100a \rightarrow b = 48 - 10a$

$f(20) = 700 \rightarrow 700 = 400a + 20b + c \rightarrow 700 = 400a + 20(48 - 10a) - 200$

$700 = 400a + 960 - 200a - 200$

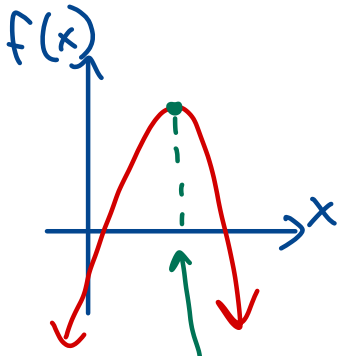
$-60 = 200a$

$a = -0.3$

$b = 48 - 10a = 51$

$$f(x) = -0.3x^2 + 51x - 200$$

(b) **[3 points]** How much should Merlin charge to maximize his profit?



he should charge this much

$$h = \frac{-b}{2a} = \frac{-51}{2(-0.3)} = 85 \text{ dollars}$$