Math 120 (Pezzoli)
Spring 2019
Midterm #2

Name ______________

TA: ______________

Section: ______________

Instructions:

- Your exam contains 3 problems.
- Your exam should contain 4 pages; please make sure you have a complete exam.
- Box in your final answer when appropriate.
- Unless stated otherwise, you **MUST** show work for credit. No credit for answers only. If in doubt, ask for clarification.
- Your work needs to be neat and legible.
- You are allowed one 8.5×11 sheet of notes (both sides). The only calculator allowed is Texas Instruments ti 30x iis.
- Round off your answers to 2 decimal places, unless you are asked for exact answers.

Problem #1(1~~5~~6 pts) ______

Problem #2(1~~5~~4 pts) ______

Problem #3(15 pts) ______

TOTAL (45 pts) ________

1. Mary has spiders and flies in her house. Yesterday she counted 5 spiders. Today she counted 8 spiders and 10 flies. The fly population triples every two days. Assuming that the number of spiders and flies in Mary's house grows exponentially, when will there be 20 times more flies than spiders in Mary's house ? Give the answer in days from today.

$f(t) = 10\left(\sqrt[2]{3}\right)^t$ fly population t days from today

$g(t) = 8\,b^t$ $\quad g(-1) = 8\,b^{-1} = 5$ $\quad b = \frac{8}{5}$

$g(t) = 8\left(\frac{8}{5}\right)^t$ spider population t days from today

want $f(t) = 20 \cdot g(t)$

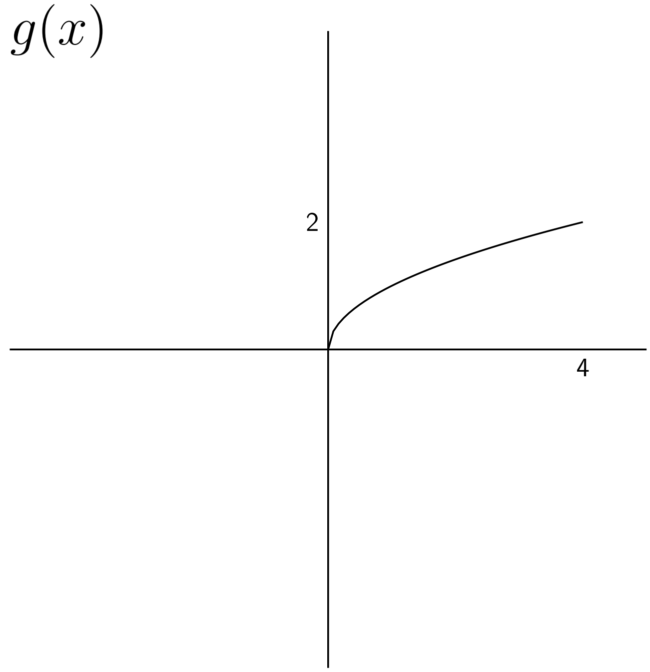
$$10\left(\sqrt[2]{3}\right)^t = 20 \cdot 8 \left(\frac{8}{5}\right)^t$$

$$\left(\frac{\sqrt[2]{3}}{\frac{8}{5}}\right)^t = 16$$

$$t = \frac{\ln 16}{\ln\left(\frac{5}{8}\sqrt[2]{3}\right)} \approx 35 \text{ days}$$

(34.96)

$$y = g(x)$$



2. Start with the function $g(x) = \sqrt{x}$ restricted to the domain $0 \leq x \leq 4$. The graph of this function is on the prevoius page. List below the graph shifts, scaling (stretches or compressions) and reflexions which, when applied to the graph of $g(x)$, in the listed order, would result in the graph of $2 - g(4+5x)$.Be precise: give the direction (right, left up or down) and the number of units for a shift, the scaling factor for scaling (ex: shift up of 3 units, or scale by a factor of 10).

HORIZONTALLY:

1): shift left 4 units The new function is : $y = \sqrt{4+x}$

2): scale by factor $\frac{1}{5}$ The new function is : $y = \sqrt{4+5x}$

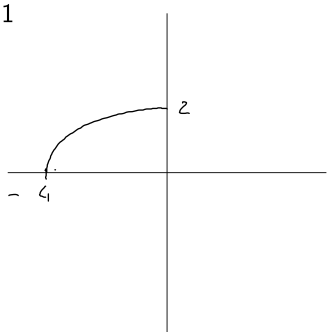
VERTICALLY:

3): reflect around x axis The new function is : $y = -\sqrt{4+5x}$

4): shift up two units The new function is: $y = 2 - \sqrt{4+5x}$

Now draw the graphs, in the same order, for the functions you obtained in the steps above. Mark the values of x intercepts (if any) on the x axis and the lowest and biggest y values and y intercepts (if any) on the y axis.

1



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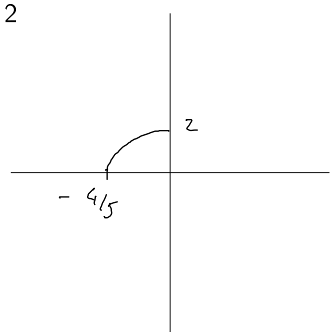

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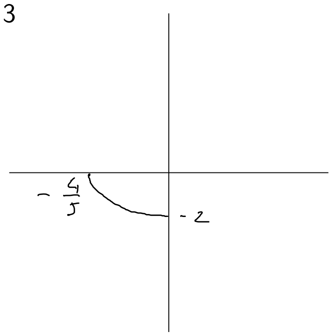

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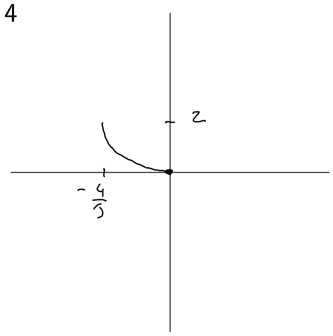

Domain $[-\frac{4}{5}, 0]$ or $-\frac{4}{5} \leq x \leq 0$

Range $[0, 2]$ or $0 \leq y \leq 2$

3. Consider the function $f(x) = 2 - \sqrt{4+5x}$. (This is similar to the function in the previous problem, but the domain of $\sqrt{x}$ is not restricted to $0 \le x \le 4$).

(a) Find the domain and range of $f(x)$.

Domain $4+5x \geq 0$ $\boxed{x \geq -\frac{4}{5}}$

Range $\sqrt{4+5x}$ could be any number ≥ 0 so $\sqrt{4+5x} \geq 0$

$-\sqrt{4+5x} \leq 0$

$2 - \sqrt{4+5x} \leq 2$

DOMAIN = $x \geq -4/5$

RANGE = $(-\infty, 2]$

(b) Compute the inverse function $f^{-1}(y)$. Show all steps. Indicate the domain for the inverse function.

$$y = 2 - \sqrt{4+5x}$$

$$\sqrt{4+5x} = 2 - y$$

$$4+5x = (2-y)^2$$

$$x = \frac{(2-y)^2 - 4}{5}$$

$$f^{-1}(y) = \frac{(2-y)^2 - 4}{5}$$

Domain $(-\infty, 2]$

(c) Compute $f(f(-\frac{4}{5}))$

$$f\left(-\frac{4}{5}\right) = 2$$

$$f(2) = 2 - \sqrt{14} \approx -1.74$$