Math 120 (Pezzoli)
Spring 2019
Midterm #1

Name _____

TA: _____

. Section: _____

Instructions:

- Your exam contains 2 problems.
- Your exam should contain 4 pages; please make sure you have a complete exam.
- Box in your final answer when appropriate.
- Unless stated otherwise, you **MUST** show work for credit. No credit for answers only. If in doubt, ask for clarification.
- Your work needs to be neat and legible.
- You are allowed one 8.5×11 sheet of notes (both sides).
- The only calculator allowed is the Ti-30x IIS.
- Round off your answers to 3 decimal places, unless you are asked for exact answers.

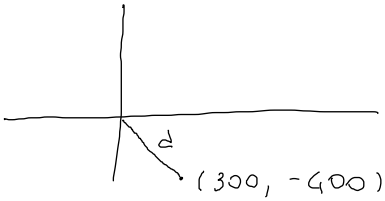
Problem #1 (20 pts) _____

Problem #2 (20 pts) _____

TOTAL (40 pts) _____

1. A United Airlines plane is flying in a straight line towards a control tower with a speed of 250 mi/hour. At time $t = 0$ it is located 300 mi East and 400 mi South of the control tower. Use a coordinate system with the origin at the control tower.

   (a) Find the parametric equations of motion for the United Airlines plane.



$d = \sqrt{300^2 + (-400)^2} = 500$

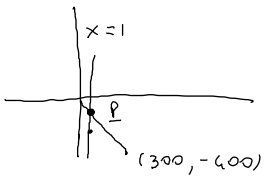
UA plane reaches control tower at $t = \frac{500}{250} = 2$

$V_x = \frac{0 - 300}{2} = -150$, $V_y = \frac{0 - (-400)}{2} = 200$

parametric equations are:

(*) $x = 300 - 150t$
$y = -400 + 200t$

   (b) An American Airlines plane is flying North at a speed of v mph. At time $t = 1$ it is located 100 mi East and 160 mi South of the control tower. It flies at the same altitude as the United Airlines plane. For which value of v do the two planes collide?



Method 1: the paths of the two planes intersect at a point $P(100, y)$ to find y 1) Plug in 100 for x in (*) $100 = 300 - 150t$

$t = \frac{200}{150} = \frac{4}{3}$ 2) $y = -400 + 200\frac{4}{3} = -\frac{400}{3}$

3) We want AA plane to be at $(100, -\frac{400}{3})$ at $t = \frac{4}{3}$

The parametric equations of AA plane are

$x = 100$
$y = -160 + v(t-1)$ so we need $-\frac{400}{3} = -160 + v(\frac{4}{3} - 1)$, $\frac{480 - 400}{3} = \frac{v \cdot 1}{3}$

$\boxed{v = 80 \text{ mph}}$

Method 2: Find equation of path of UA plane $y = -\frac{4}{3}x$
Find equation of path of AA plane: $x = 100$. Find intersection $P(100, -\frac{400}{3})$, find time UA plane reaches P $\frac{\sqrt{(-\frac{400}{3} + 400)^2 + (100 - 300)^2}}{250} = \frac{4}{3}$
to reach P at time $t = \frac{4}{3}$ AA plane needs to cover a distance of $\sqrt{(-\frac{400}{3} + 160)^2 + 0^2} = \frac{80}{3}$ in $t = \frac{4}{3} - 1 = \frac{1}{3}$ so $v = \frac{\frac{80}{3}}{\frac{1}{3}} = 80$

(problem 1 continued)

(c) If the speed of the American Airlines plane is 240 mph (instead of the value v you found in part (b)), when are the two planes closest? How close do they get?

$$d(t) = \sqrt{(300-150t-100)^2 + (-400+200t-(-160t+240(t-1)))^2}$$

$$= \sqrt{(200-150t)^2 + (-40t)^2}$$

$$= \sqrt{(150^2+40^2)t^2 - 400\cdot 150t + 150^2}$$

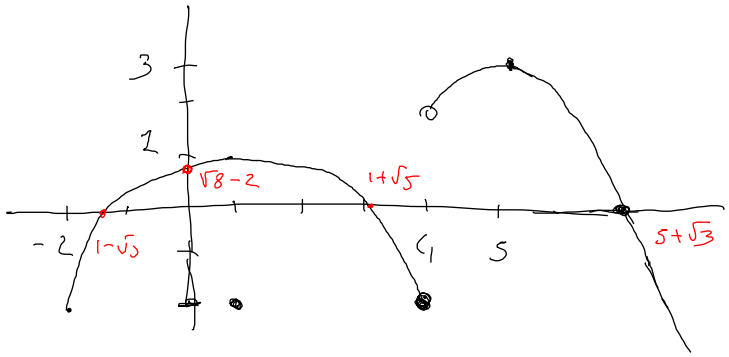
planes are closest when $t = \dfrac{400\cdot 150}{2(150^2+40^2)} = \dfrac{300}{241} \approx 1.245$ h

$d\left(\dfrac{300}{241}\right) \approx$ is the closest distance

2. The function $f(x)$ is defined as follows :

$$f(x) = \begin{cases} \sqrt{9-(x-1)^2} - 2, & \text{if } -2 \le x \le 4 \\ 3-(x-5)^2, & \text{if } x > 4 \end{cases}$$

(a) Draw the graph of the function .



(b) Find the y intercept, and mark it on the graph you drew in part (a).

$f(0) = \sqrt{9-(0-1)^2} - 2 = \sqrt{8} - 2 \approx .028$

c) Find the range of f

$(-\infty, 3]$

(d) Find the x intercepts and mark them on the graph you drew in part (a).

$\sqrt{9-(x-1)^2} - 2 = 0$

$\sqrt{9-(x-1)^2} = 2$

$9-(x-1)^2 = 4$

$5 = (x-1)^2$

$\pm\sqrt{5} = x-1$

$1 \pm \sqrt{5} = x$

$3-(x-5)^2 = 0$

$3 = (x-5)^2$

$\pm\sqrt{3} = x-5$

$5+\sqrt{3} = x$