Math 120 (Pezzoli)
Spring 2015
Midterm #2

Name ________

TA: ________

Section: ________

Instructions:

- Your exam contains 4 problems.
- Your exam should contain 5 pages; please make sure you have a complete exam.
- Box in your final answer when appropriate.
- Unless stated otherwise, you **MUST** show work for credit. No credit for answers only. If in doubt, ask for clarification.
- Your work needs to be neat and legible.
- You are allowed one 8.5×11 sheet of notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed.
- Round off your final answers to 2 decimal places, unless you are asked for exact answers.

Problem #1 (9 pts) ________

Problem #2 (13 pts) ________

Problem #3 (13 pts) ________

Problem #4 (15 pts) ________

TOTAL (50 pts) ________

1. Knotweed is an invasive plant. In the year 2000 there were 100 knotweeds in Greenforest. In 2002 the number of knotweeds in Greenforest had grown to 150 , and there were 200 knotweed plants in Greenforest in 2005. Assume that the number of knotweed plants in Greenforest can be modeled by a linear to linear function .

   (a) Let f(t) be the function defined on $0 \leq t \leq \infty$, giving the number of knotweeds in Greenforest t years after 2000. Find a formula for f(t).

$f(t) = \dfrac{at+b}{t+c}$

3 pt for setting up

$\begin{cases} 100 = b/c \\ 150 = \dfrac{2a+b}{2+c} \\ 200 = \dfrac{5a+b}{5+c} \end{cases}$ $\begin{cases} 100c = b \\ 300 + 150c = 2a + b \\ 1000 + 200c = 5a + b \end{cases}$ $\begin{cases} 100c = b \\ 300 + 150c = 2a + 100c \\ 1000 + 200c = 5a + 100c \end{cases}$

3 points for solving

$\begin{cases} 100c = b \\ 150 + 25c = a \\ 1000 + 100c = 750 + 125c \end{cases}$ $\begin{cases} c = 10 \\ b = 1000 \\ a = 600 \end{cases}$ $f(t) = \dfrac{400t + 1000}{t + 10}$

   (b) In the long run how many knotweeds will there be in Greenforest, according to this model ?

long term value is $a = 400$ 2 pt

   (c) Find a formula for $f^{-1}(w)$, the inverse of f(t), an~~d give the domain and range of the inverse~~.

$w = \dfrac{at+b}{t+c}$ $wt + wc = at + b$ $(w-a)t = b - wc$

$t = f^{-1}(w) = \dfrac{b - wc}{w - a} = \dfrac{1000 - 10w}{w - 400}$ 4 pt

   (d) Calculate $f^{-1}(250)$ and explain in words what is the meaning of this value.

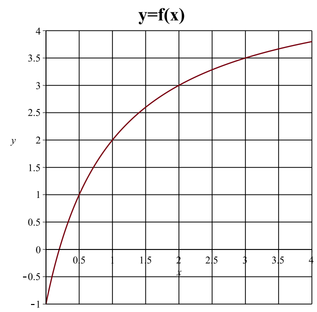
$f^{-1}(250) \approx 10$ 2 pt

it is the number of years i e for

the number of knotweeds to g 250

1 pt

2. Below you are given the graph of f(x) . The domain of f(x) is $0 \leq x \leq 4$ and the range is $-1 \leq y \leq 3.8$



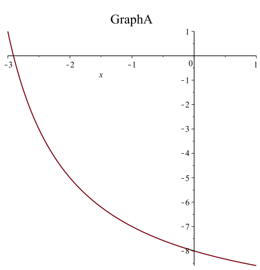
a) What is the value of f(f(2)) ?

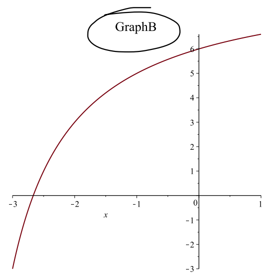
$f(f(2)) = f(3) = 3.5$ 3 pt

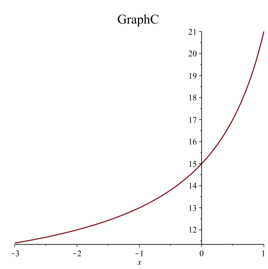
b) What is the value of $f^{-1}(2)$?

$f^{-1}(2) = 1$ 3 pt

c) Which of the following is the graph of 2f(x+3)-1 ? Circle the correct graph. 3pt







d) Find the domain of $\frac{f^{-1}(2x+1)}{x-1}$

Domain of f^{-1} is equal to range f

$-1 \leq x \leq 3.8$ 2 pt so we want

$-1 \leq 2x+1 \leq 3.8$ $x \neq 1$ 2 pt $-1 \leq x \leq 1.4$ $x \neq 1$ or

$[-1 \ \ 1)$ and $(1, 1.4]$ 3

3. Town A population triples every 100 years. City B population doubles every 90 years. In the year 2000 the two cities have the same population of 10000. When will city A have twice as many inhabitants as city B ?

$A(t)$ = population of Town A $\quad A(t) = 10000\,a^t$

$30000 = 10000\,a^{100} \quad a = \sqrt[100]{3} \quad A(t) = 10000\left(\sqrt[100]{3}\right)^t$ 4 pt

$B(t)$ = population of town B $\quad B(t) = 10000\,b^t$

$20000 = 10000\,b^{90} \quad b = \sqrt[90]{2} \quad B(t) = 10000\left(\sqrt[90]{2}\right)^t$

4 pt

$A(t) = 2B(t)$ if

$10000\left(\sqrt[100]{3}\right)^t = 20000\left(\sqrt[90]{2}\right)^t$ 2 pt

$$\left(\frac{\sqrt[100]{3}}{\sqrt[90]{2}}\right)^t = 2 \qquad t = \frac{\ln 2}{\ln\left(\frac{\sqrt[100]{3}}{\sqrt[90]{2}}\right)} \approx 211.04$$

3 pt

4. The cost $c(x)$ of making x items is $c(x) = 2x^2 - 200x + 15000$. How many items do you need to make in order to minimize the cost ?

$x = \frac{200}{4} = 50$ 3 pt

Suppose that the revenue $r(x)$ produced by selling x of the above items is also modeled by a quadratic function. The revenue produced by selling 0 items is of course 0, and the revenue is at a maximum of \$ 30,000 when 100 items are sold. How many items x should be made and sold in order to maximize the profit $p(x) = r(x) - c(x)$ (revenue - cost)?

$r(x) = a(x-100)^2 + 30{,}000$ 3 pt

$0 = a \cdot 10000 + 30000 \quad a = -3$

$r(x) - c(x) = -3(x^2 - 200x + 10000) + 30000$
$- 2x^2 + 200x - 15000 =$ 1 pt

$-5x^2 + 800x - 15000$

has max at $x = \frac{-800}{-10} = 80$

2 pt