Math 120 (Pezzoli)
Spring 2015
Midterm #1

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Name ______________

TA: ______________

Section: ______________

Instructions:

- Your exam contains 3 problems.
- Your exam should contain 5 pages; please make sure you have a complete exam.
- Box in your final answer when appropriate.
- Unless stated otherwise, you **MUST** show work for credit. No credit for answers only. If in doubt, ask for clarification.
- Your work needs to be neat and legible.
- You are allowed one 8.5×11 sheet of notes (both sides). Graphing calculators are NOT allowed; scientific calculators are allowed.
- Round off your final answers to 2 decimal places, unless you are asked for exact answers.

Problem #1 (15 pts) ______

Problem #2 (20 pts) ______

Problem #3 (15 pts) ______

TOTAL (45 pts) ________

1. (a) Write the equation of the line through P (1,0) and parallel to the line $5x + 2y - 1 = 0$

$y = -\frac{5}{2} x + \frac{1}{2}$ The line we want has the form $y = -\frac{5}{2} x + b$ To find b set

$0 = -\frac{5}{2} \cdot 1 + b$ so $b = \frac{5}{2}$

$\boxed{y = -\frac{5}{2} x + \frac{5}{2}}$

or $\boxed{y = -\frac{5}{2}(x-1)}$

(b) Write the equation of the line through Q(3,1) and tangent to the circle of radius $\sqrt{5}$ centered at (1,0)= O

$(x-1)^2 + y^2 = 5$, $(3-1)^2 + 1^2 = 5$ Therefore Q is on the circumference.

Find slope of line OP : $\frac{1-0}{3-1} = \frac{1}{2}$

our tangent has equation $y = -2x + b$

to find b solve $1 = -2 \cdot 3 + b$, so $b = 7$

$\boxed{y = -2x + 7}$ or $\boxed{y - 1 = -2(x-3)}$

2. Tom's house is 10 miles West and 4 miles South of Bob's house. Mary's house is 2 miles East and 1 mile North of Bob's house. At time $t = 0$ Tom starts jogging from his house to Mary's house. Tom's speed is 6 miles/hour.

(a) Write parametric equations for Tom's position with respect to Bob's house t hours into his jog.

$t = 0$ at $(-10, -4)$

$t = ?$ at $(2, 1)$

$13/6$

d(Tom's house, Mary's house) $= \sqrt{25 + 144} = 13$, time Tom reaches Mary's house is $\frac{13}{6}$ hour

$x = at + c \qquad y = ct + d$

$-10 = c \qquad -4 = d$

$2 = a \cdot \frac{13}{6} + c \qquad 1 = c \cdot \frac{13}{6} + d$

$c = -10, \; a = \frac{72}{13}, \; d = -4, \; c = \frac{30}{13}$

so $\boxed{\left(\frac{72}{13}t - 10, \; \frac{30}{13}t - 4\right)}$

(b) What is Tom's position, with respect to Bob's house, 20 min into his jog ?

$\frac{72}{13} \cdot \frac{1}{3} - 10 \approx -8.15 \qquad \frac{30}{13} \cdot \frac{1}{3} - 4 \approx -3.23$

so $(-8.15, -3.23)$ or 8.15 miles West and 3.23 miles South

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(c) Bob is baking a cake and you can smell the cake when you are within 0.5 miles of Bob's house. For how long during his jog from his house to Mary's house does Tom smell the cake ?

$$x^2 + y^2 = \frac{1}{4}$$

$$\left(\frac{73}{13}t - 10\right)^2 + \left(\frac{30}{13}t - 4\right)^2 = \frac{1}{4}$$ solving for

t gives us the times when Tom starts and stops smelling the cake:

$$\left(\frac{73^2}{13^2} + \frac{30^2}{13^2}\right)t^2 - \left(2 \cdot \frac{73}{13} \cdot 10 + 2 \cdot \frac{30}{13} \cdot 4\right)t + 100 + 16 - \frac{1}{4} = 0$$

$$36.8580\, t^2 - 130.7692\, t + 115.75 = 0$$

$$t \approx 1.8546 \text{ and } 1.6933$$

so $1.8546 - 1.6933 \approx \boxed{0.16 \text{ hr}}$

Alternetive solution

Tom's path: $\frac{y+4}{x+10} = \frac{1+4}{2+10}$

or $y = \frac{5}{12}x + \frac{2}{12}$

solve $\begin{cases} y = \frac{5}{12}x + \frac{2}{12} \\ x^2 + y^2 = \frac{1}{4} \end{cases}$

$x \approx 0.38, -0.4983$
$y \approx 0.325, -0.041$

Tom's sterts smelling the cake at $P(-0.4983, -0.041)$ and stops at $Q(0.38, 0.325)$

$$d(PQ) = 0.9496$$

$$t = \frac{d}{v} = \frac{0.9496}{6} \approx$$

$$\boxed{0.16}$$

3. John invested \$5,000 in the year 2000. Let v(t) be the value of John's investment t years after 2000. Assume that for $0 \leq t \leq 10$, v(t) can be modeled by a parabola, using the following information: in the first eight years after his initial investment John's money grew to a maximum of \$10,000 in 2008 and then dropped in value from 2008 to 2010. For the next 4 years, from 2010 to 2014, John's investment increased at a constant rate of \$1,200 a year. Let v(t) be the value of John's investment t years after 2000. Write a formula for $v(t)$ valid for $0 \leq t \leq 14$

for $0 \leq t \leq 10$

$v(t) = a(t-8)^2 + 10{,}000$ to find a set

$5000 = a(0-8)^2 + 10\,000 \qquad a = \frac{-5000}{64} = -78.125$

$v(10) = -78.125(10-8)^2 + 10{,}000 = 9687.5$

for $10 \leq t \leq 14$ $v(t)$ is linear

$v(t) = 1200t + b$ to find b set

$\underbrace{9687.5}_{v(10)} = 1200 \cdot \underbrace{10}_{t=0} + b \;, \quad b = -2312.5$

$$\text{so } v(t) = \begin{cases} -78.125(t-8)^2 + 10{,}000 & \text{if } 0 \leq t \leq 10 \\ 1200t - 2312.5 & \text{if } 10 < t \leq 14 \end{cases}$$

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When in the time range between the years 2000 and 2015 was John's investment worth \$8000 ? (give your answer as a decimal number).

Solve $-78.125(t-8)^2 + 10\,000 = 8000$

$(t-8) = \pm\sqrt{\dfrac{2000}{78.125}}$

$t = 8 \pm \sqrt{\dfrac{2000}{78.125}} \approx 13.06$ and 2.94

So $\boxed{t = 2.94}$

Since $v(10) > 8000$ for $10 \le t \le 14$ $v(t) > 8000$