Math 120 Section A, Spring 2014
Midterm Exam Number Two: Solutions

1. (a) Start by multiplying both sides by x to get $x^2 + 3x = 15 + x$. Rearrange to find $x^2 + 2x - 15 = 0$, so $x = -5$ or $x = 3$ by the quadratic formula (or factoring).

   (b) Divide by 2000 to get $2.5 = x^9$, then take the ninth root of both sides. $x = \sqrt[9]{2.5}$, which is about 1.107.

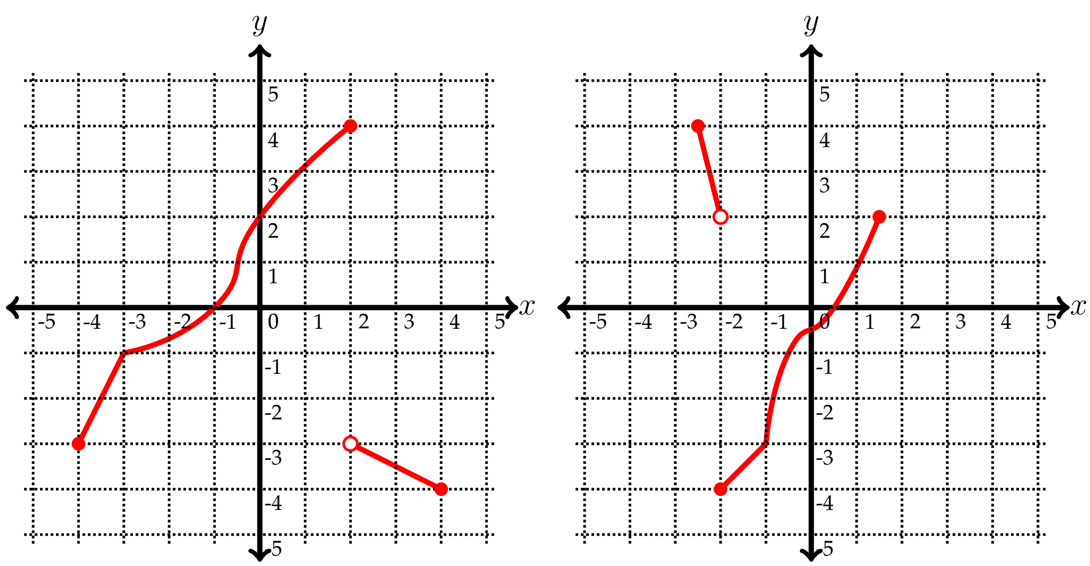
   (c) Take the natural log of both sides to get $5x = \ln(12)$, so $x = \ln(12)/5 \approx 0.497$.

   (d) If $\ln(3x + 2) = 4$, then $3x + 2 = e^4$, so $x = (e^4 - 2)/3 \approx 17.533$.

2. (a) $f(f(f(2))) = f(f(0)) = f(-1) = -3$, by reading the graph.

   (b) The graph of $f^{-1}(x)$ is the graph of $f(x)$, reflected over the line $y = x$, as shown on the left.

   (c) The graph of $f(2x + 1)$ is the graph of $f(x)$, shifted left by one and then scaled horizontally by a factor of $1/2$, as shown on the right.



3. Draw a table like this. v is measured in inches per second, ω in radians per second, and r in inches.

Wheel	v	ω	r
Wheel A		$2\pi/20$	8
Wheel B			5
Wheel C			2
Wheel D		?	9

Then, fill out the rest of the table by using $v = \omega r$ and matching values that share a box.

Wheel	v	ω	r
Wheel A	$4\pi/5$	$2\pi/20$	8
Wheel B	$4\pi/5$	$4\pi/25$	5
Wheel C	$8\pi/25$	$4\pi/25$	2
Wheel D	$8\pi/25$	$8\pi/225$	9

So Wheel D has an angular speed of $8\pi/225$ radians per second. Multiply by 60 and divide by 2π to get $240/225 = 48/45 \approx 1.0667$ RPM.

4. Okay, linear-to-linear rational function time. We want a function $f(x) = \dfrac{ax+b}{x+d}$ satisfying three criteria: $f(3) = 120$, $f(6) = 108$, and the horizontal asymptote is $y = 60$. This leads to three equations:

$$\frac{3a+b}{3+d} = 120 \qquad \frac{6a+b}{6+d} = 108 \qquad a = 60$$

Plugging the third equation into the others and simplifying gives

$$180 + b = 360 + 120d \qquad 360 + b = 648 + 108d$$

Subtract the first equation from the second to get $180 = 288 - 12d$, so $d = 9$. Finally, plug that back into one of the other equations and solve for b to get $b = 1260$.

So the equation is $f(x) = \dfrac{60x + 1260}{x+9}$, and we want to find x so that $f(x) = 70$. From here the algebra is pretty straightforward and we end up with $x = 63$ minutes.

5. (a) First attempt: the area of a sector with radius r and angle θ is $\frac{1}{2}r^2\theta$.

   Oh, darn, that includes θ. Let's use the fact that the total length of the fence is 100. What's the total length? Well, that's $r + r + r\theta$, so $2r + r\theta = 100$, so $\theta = (100-2r)/r$. Now we can plug that back in to get $A = \frac{1}{2}r^2\left(\frac{100-2r}{r}\right)$, which we can simplify to $A = -r^2 + 50r$.

   (b) We want to know the radius r that gives the maximum value to $A = -r^2 + 50r$. That's $h = \dfrac{-b}{2a} = \dfrac{-50}{2(-1)} = 25$.