Math 120 Section A, Spring 2014
Midterm Exam Number One: Solutions

1. (a) Bevers begins at $(5, -2)$ and we have $\Delta x = 1$, $\Delta y = 15$, and $\Delta t = 5$, so:

$$x = 5 + \frac{1}{5}t \qquad y = -2 + \frac{15}{5}t$$

(b) Lincoln begins at $(3.5, 7)$ and we have $\Delta x = 4.5$, $\Delta y = -2.4$. But we don't know Δt! Instead, we need to find out how far Lincoln traveled: the distance from $(3.5, 7)$ to $(8, 4.6)$ is $d = \sqrt{(3.5 - 8)^2 + (7 - 4.6)^2} = 5.1$, and he walked at a speed or 3 units per second, so it took him $\Delta t = 5.1/3 = 1.7$ seconds.

$$x = 3.5 + \frac{4.5}{1.7}t \qquad y = 7 + \frac{-2.4}{1.7}t$$

2. We need to solve three equations:

- $x = 2x$ has one solution, $x = 0$, but this doesn't satisfy the inequality $x \le -2$.
- $x = 4x^2 + 2x - 14$ has two solutions, $x = -2$ and $x = 1.75$, but only $x = 1.75$ satisfies the inequality $-2 < x < 2$.
- $x = 6$ satisfies $x \ge 2$.

So, in total, we have two fixed points: $x = 1.75$ and $x = 6$.
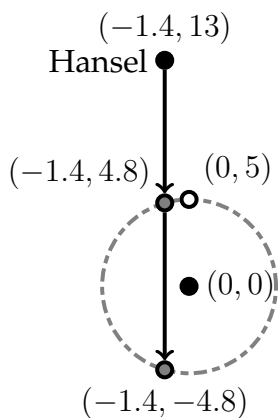
3. Here's a picture. Hansel starts at $(-1.4, 13)$ and begins walking south along the line $x = -1.4$ until he intersects the circle $x^2 + y^2 = 5^2$. To find the y-coordinates of the points of intersection, we plug in $x = -1.4$ to the equation of the circle and get $y = \pm 4.8$.

For the 8.2 kilometers that he walks south until he hits the forest, Hansel's speed is 4.5 kilometers per hour, so that part takes him $8.2/4.5$ hours. Then he walks 9.6 kilometers through the forest at a speed of 2.5 kilometers per hour, which takes him another $9.6/2.5$ hours. Finally, we convert to minutes to get a total time of $(8.2/4.5 + 9.6/2.5) \times 60 \approx 339.7$ minutes.

4. Here's a picture. The line from Dido's house to Circe's house is $y = (1/2)x$, so the perpendicular line through Helen's house has slope -2, so its equation is $y = -2(x-5)+6$. To find the intersection of these lines, we solve the equation $(1/2)x = -2(x-5)+6$ to get $x = 6.4$, $y = 3.2$, so that's where Dido turns towards Helen's house.
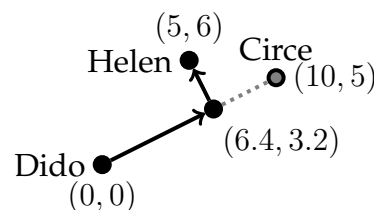
Therefore, the total distance that Dido walks is $\sqrt{6.4^2 + 3.2^2} + \sqrt{1.4^2 + 2.8^2} \approx 10.286$ miles.

5. In each of the two cases, we find the area of the shaded piece:
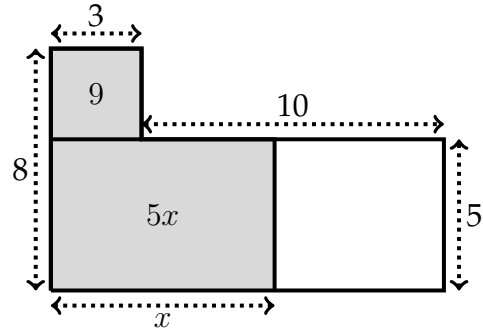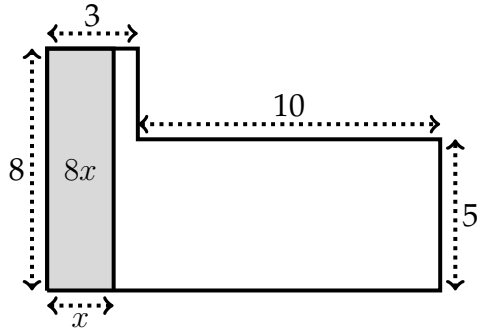
The first picture is what it looks like when x is between 0 and 3, and the second is when x is between 3 and 13. So we have:

$$f(x) = \begin{cases} 8x & \text{if } 0 \le x \le 3 \\ 9 + 5x & \text{if } 3 \le x \le 13 \end{cases}$$