1. (15 pts) Consider the function $f(x) = 3 - \sqrt{2x - 1}$.

a) (7 pts) Compute the inverse function, $f^{-1}(y)$. Show all steps. Indicate the correct domain for the inverse function.

$$y = 3 - \sqrt{2x-1}$$
$$y - 3 = -\sqrt{2x-1}$$
$$(y-3)^2 = 2x - 1$$
$$2x = (y-3)^2 + 1$$
$$\boxed{f^{-1}(y) = \frac{(y-3)^2 + 1}{2}}$$
$$\boxed{= \frac{y^2 - 6y + 10}{2}}$$
$$\boxed{= \frac{1}{2}y^2 - 3y + 5}$$

domain for f^{-1} = range of f is $\boxed{y \le 3}$

(because $y = 3 - \underbrace{\sqrt{2x-1}}_{\text{positive or zero}}$)

b) (8 pts) In this part, there is no need to show work or justify your answers. Start with the basic function

$g(x) = \sqrt{x}$, restricted to domain $0 \le x \le 4$.

List the graph shifts, stretches or compressions, and reflections which, when applied to the graph of $g(x) = \sqrt{x}$ in the listed order, would result in the graph of $f(x) = 3 - \sqrt{2x - 1}$. Be precise (ex: "first a shift up by 2 units").

Horizontally: First SHIFT RIGHT 1 UNIT

Then COMPRESS BY A FACTOR OF 2 ($\times \frac{1}{2}$)

OR

COMPRESS $\times \frac{1}{2}$

SHIFT RIGHT by $\frac{1}{2}$

Vertically: First REFLECTION IN x-axis

Then SHIFT UP 3 UNITS.

If we start with domain $0 \le x \le 4$ for $g(x) = \sqrt{x}$, what is the resulting domain after the above transformations?

$1/2 \le x \le 5/2$

$\boxed{2}$ (7 pts) Consider the function:

$$\mathbf{y = (-0.5)8^{(2+(x/3))}}$$

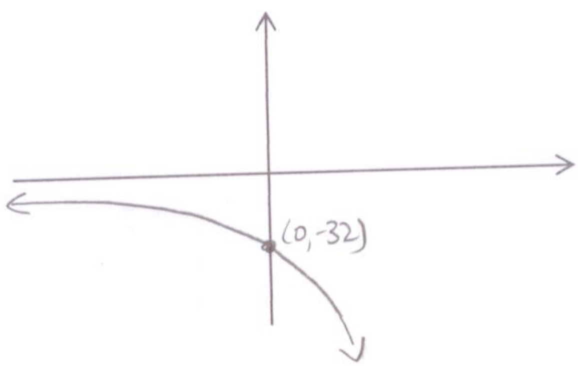
Put it in standard exponential form, $y = Ab^x$. Show your steps, and box your final answer.

$$y = (-0.5)(8^2)(8^{x/3})$$

$$= \underbrace{(-0.5)(64)}\,(8^{1/3})^x$$

$$= -32\,(\sqrt[3]{8})^x \quad \Rightarrow \quad \boxed{y = -32(2)^x}$$

Then sketch its graph, and label the values of any x or y-intercepts. No need to show work.



$\boxed{3}$ (5 pts) Solve the equation: $10^{\log_3 x} = 7$

Apply $\log_{10}$ to both sides:

$$\log_{10}\left(10^{\log_3 x}\right) = \log_{10} 7$$

$$\log_3 x = \log_{10} 7$$

$$3^{\log_3 x} = 3^{\log_{10} 7}$$

$$\boxed{x = 3^{\log_{10} 7}}$$

OR

Apply $\ln(\ )$ to both sides

$$\ln\left(10^{\log_3 x}\right) = \ln(7)$$

$$\log_3 x \ \ln(10) = \ln(7)$$

$$\log_3 x = (\ln 7)/\ln(10)$$

$$\boxed{x = 3^{\frac{\ln(7)}{\ln(10)}}}$$

$$\boxed{x \cong 2.53}$$

4 (13 pts) In 1990, the U.S. minimum wage was \$3.80 per hour. In 1997, it was \$5.15 per hour. Assume the minimum wage grows according to an exponential model $W(t)$, where t represents the number of years after 1990.

a) (6 pts) Find a formula for $W(t)$.

$W(t) = W_0 b^t$

(1990) $t=0$: * $W_0 = W(0) = 3.80$

(1997) $t=7$ * $W(7) = 3.80\, b^7 = 5.15$

$b^7 = \frac{5.15}{3.80}$

$b = \sqrt[7]{\frac{515}{380}}$

$$\boxed{W(t) = 3.80\left(\sqrt[7]{\frac{515}{380}}\right)^t = 3.80(1.0443847)^t}$$

b) (2 pts) What does the model predict for the current minimum wage? (year 2012)

$t = 2012 - 1990 = 22$

$W(22) = 3.80\left(\sqrt[7]{\frac{515}{380}}\right)^{22} \cong \boxed{\$9.88}$

c) (5 pts) In what year is the minimum wage expected to reach \$100 per hour, according to this model?

$100 = 3.80\left(\sqrt[7]{\frac{515}{380}}\right)^t$

$\frac{100}{3.80} = \left(\sqrt[7]{\frac{515}{380}}\right)^t$

$\ln\left(\frac{100}{3.80}\right) = t \ln\left(\sqrt[7]{\frac{515}{380}}\right)$

$t = \frac{\ln\left(\frac{100}{3.80}\right)}{\ln\left(\sqrt[7]{\frac{515}{380}}\right)} = \frac{\ln\left(\frac{100}{3.80}\right)}{\frac{1}{7}\ln\left(\frac{515}{380}\right)} \cong \frac{3.270169\ldots}{0.043427\ldots}$

$\cong 75.3$ years after 1990, so in year $\boxed{2065}$

5 (10 pts) The number of doughnuts sold by a local shop depends on the amount of money spent on advertising. If the shop spends \$50, it sells an average of 200 doughnuts a day. If it spends \$200 in advertising, it sells 350 doughnuts a day. As the shop spends more and more on advertising, the number of doughnuts sold will approach (but not reach) 600 doughnuts per day.

Assume that the number y of doughnuts sold per day is a linear-to-linear rational function of the $\$x$ spent on advertising.

Determine how much money the shop should spend on advertising in order to sell 500 doughnuts per day.

$$f(x) = \frac{ax+b}{x+d}$$

$x = \$$ spent
$f(x) = \#$ doughnuts sold.

$f(50)=200$: $\dfrac{50a+b}{50+d} = 200 \Rightarrow \boxed{50a+b = 10{,}000+200d}$ (1)

$f(200)=350$: $\dfrac{200a+b}{200+d} = 350 \Rightarrow \boxed{200a+b = 70{,}000+350d}$ (2)

as $x \to \infty$ $f(x) \to a$, so $\boxed{a = 600}$ (3)

Replacing $a=600$ into (1) & (2), and solving for b in each

(1) $\Rightarrow$ $b = 10{,}000 + 200d - 50(600) = 200d - 20{,}000$

(2) $\Rightarrow$ $b = 70{,}000 + 350d - 200(600) = 350d - 50{,}000$

Set equal: $200d - 20{,}000 = 350d - 50{,}000$

Solve for d: $30{,}000 = 150d$

$d = 200$

From b $= 200d - 20{,}000 = 200(200) - 20{,}000 = 20{,}000$

so $\boxed{f(x) = \dfrac{600x + 20{,}000}{x + 200}} = 500$

$600x + 20{,}000 = 500(x+200)$

$600x + 20{,}000 = 500x + 100{,}000$

$100x = 80{,}000$

$x = 800$ so spend \$800