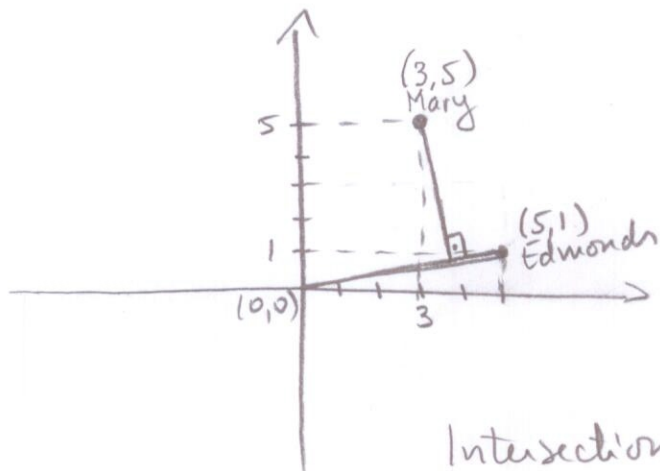


1. (14 pts) **Mary's sailboat** is sitting still at a location **5 miles North and 3 miles East of the city of Kingston**.
 A ferry has just left Kingston and is traveling in a straight line to Edmonds.
Edmonds is located 1 mile North and 5 miles East of Kingston.

- a) (8 pts) Draw a picture and compute where the ferry will be located when it is closest to Mary's sailboat.



Kingston - Edmonds line: $y = \frac{1}{5}x$

Perpendicular from Mary's location:

$$y = -5(x-3) + 5$$

$$= -5x + 20$$

Intersection point: $\frac{1}{5}x = -5x + 20$

$$\frac{26}{5}x = 20$$

$$x = \frac{100}{26} = \frac{50}{13} \approx 3.85$$

$$y = \frac{1}{5}\left(\frac{50}{13}\right) = \frac{10}{13} \approx 0.77$$

ANSWER: Ferry will be 3.85 miles East and 0.77 miles North of Kingston

- b) (6 pts) Suppose the ferry travels at a uniform speed of 14 feet per second. How long does the ferry take to reach Edmonds? Give your answer in minutes.

$$\text{speed} = 14 \frac{\text{ft}}{\text{sec}} \times \frac{60 \text{ sec}}{1 \text{ min}} \times \frac{1 \text{ mile}}{5280 \text{ ft}} = \frac{840 \text{ miles}}{5280 \text{ min}} \approx 0.15909 \frac{\text{miles}}{\text{minute}}$$

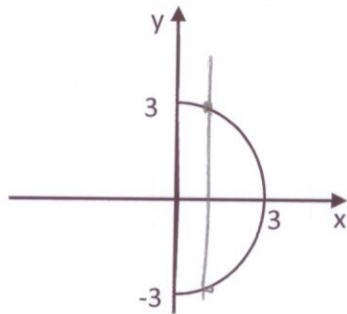
(OR 9.54 mph)

$$\text{distance } d = \sqrt{(5-0)^2 + (1-0)^2} = \sqrt{26} \text{ miles} \approx 5.099 \text{ miles}$$

$$\text{time} = \frac{\sqrt{26} \text{ miles}}{0.15909 \text{ miles/min}} \approx 32.05 \text{ min}$$

ANSWER: 32.05 minutes

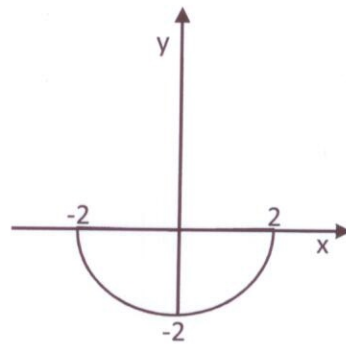
2. (8 pts) For each box (A)-(D) below, determine if the given graph establishes a function relationship, with y a function of x . There is no need to show work or justify your answers..



(A) Is this a function? Circle YES or NO

If YES, list its domain: _____

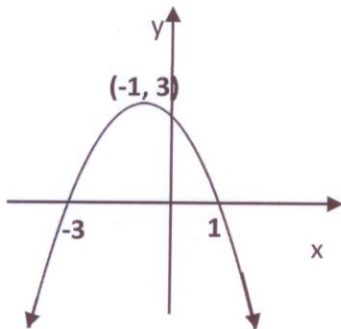
& range: _____



(B) Is this a function? Circle YES or NO

If YES, list its domain: $-2 \leq x \leq 2$

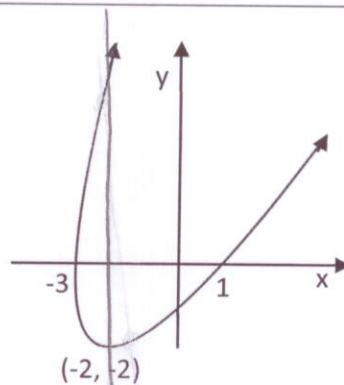
& range: $-2 \leq y \leq 0$



(C) Is this a function? Circle YES or NO

If YES, list its domain: \mathbb{R}

& range: $y \leq 3$



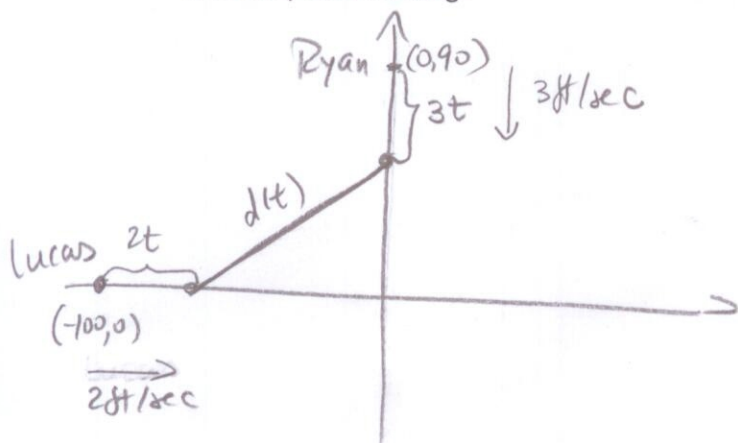
(D) Is this a function? Circle YES or NO

If YES, list its domain: _____

& range: _____

3. (12 pts) Ryan starts walking due South at 3 feet per second from a point 90 feet North of an intersection. At the same time, Lucas starts walking due East at 2 feet per second from a point 100 feet West from the intersection.

a) (6 pts) Draw a picture and write an expression in t for the distance $d(t)$ between Ryan and Lucas t seconds after they start walking.



$$\text{Ryan: } (x_R, y_R) = (0, 90 - 3t)$$

$$\text{Lucas: } (x_L, y_L) = (-100 + 2t, 0)$$

$$d = \sqrt{(x_R - x_L)^2 + (y_R - y_L)^2}$$

$$\text{ANSWER: } d(t) = \sqrt{(100 - 2t)^2 + (90 - 3t)^2}$$

b) (6 pts) When are Ryan and Lucas closest? What is the shortest distance between them?

$$d(t) = \sqrt{10,000 - 400t + 4t^2 + 8100 - 540t + 9t^2}$$

$$= \sqrt{13t^2 - 940t + 18,100}$$

smallest value of $13t^2 - 940t + 18,100$ is at

$$\text{vertex: } t = \frac{-(-940)}{2(13)} = \frac{940}{26} = \frac{470}{13} \approx 36.1538$$

Shortest distance is

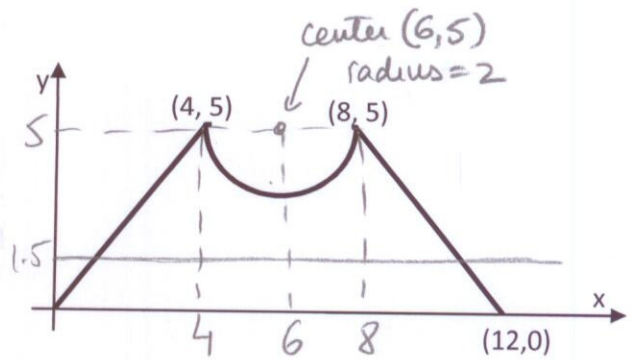
$$d(36.1538) \approx \sqrt{13(36.1538)^2 - 940(36.1538) + 18,100}$$

$$\approx \sqrt{1107.6923} \approx 33.28$$

Answer: They're closest at 36.15 seconds, when they are 33.28 feet apart.

4. (16 pts) The vertical cross-section of a toy volcano is given in the picture. The coordinates of certain points are listed on the picture. All units are in inches

Assume the sides of the volcano are line segments, and the pit of the volcano is the lower half of a circle.



- a) (7 pts) Find a multipart function $y = f(x)$ that models the vertical cross-section of this volcano. (No need to show work, but write your answer in correct bracket notation for multipart functions.)

$$y = \begin{cases} \frac{5}{4}x & \text{if } 0 \leq x \leq 4 \\ 5 - \sqrt{4 - (x-6)^2} & \text{if } 4 \leq x \leq 8 \\ -\frac{5}{4}(x-12) & \text{if } 8 \leq x \leq 12 \end{cases}$$

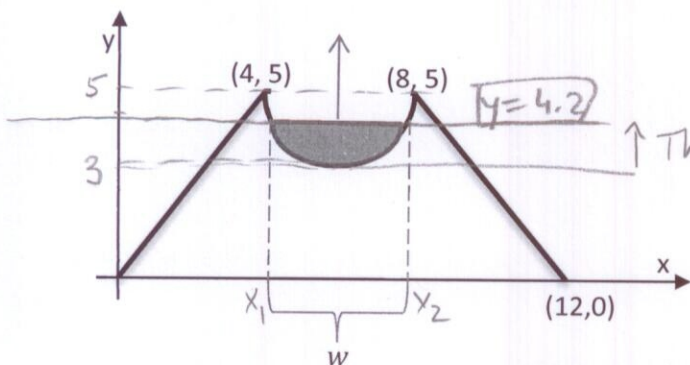
- b) (6 pts) Solve the equation $f(x) = 1.5$, where $f(x)$ is the function modeling the cross-section of the volcano.

Line $y=1.5$ only crosses the sides of the volcano:

① $\frac{5}{4}x = 1.5$
 $x = \frac{6}{5} = 1.2$

② $-\frac{5}{4}(x-12) = 1.5$
 $x-12 = -\frac{6}{5} = -1.2$
 $x = 12 - 1.2 = 10.8$
 $x = 10.8$

- c) (5 pts) Suppose the pit of the volcano is filling with fake lava, and the depth of the lava in the pit rises at a rate of 0.02 inches per second. What is the width w of the filled portion of the pit after 1 minute?



↑ This is $(0.02 \text{ in/sec}) \times (60 \text{ sec}) = 1.2 \text{ in}$ after 1 min.
 Bottom is $y=3$, so the level line at 1 minute is $y = 3 + 1.2 = 4.2$

Intersect $y=4.2$ with circle: $(x-6)^2 + (y-5)^2 = 4$

$$(x-6)^2 + (4.2-5)^2 = 4$$

$$(x-6)^2 = 4 - 0.64 = 3.36$$

$$x-6 = \pm \sqrt{3.36} \Rightarrow x = 6 \pm \sqrt{3.36}$$

$$w = x_2 - x_1 = (6 + \sqrt{3.36}) - (6 - \sqrt{3.36}) = 2\sqrt{3.36} \approx 3.666\dots$$

$$w \approx 3.67 \text{ in}$$