Math 120 A - Spring 2006
Mid-Term Exam Number One
April 20, 2006
Hints and Answers

1. This problem can be solved using methods that were used to solve problem 6.4 of the text, to which it is identical except for the dimensions.

   Version 1 (the pizza is 9 by 21 inches):

   $$A(x) = \begin{cases} \frac{9}{30}x^2 & \text{if } 0 \le x \le 15, \\ 94.5 - \frac{9}{12}(x-21)^2 & \text{if } 15 \le x \le 21 \end{cases}$$

   Version 2 (the pizza is 7 by 21 inches):

   $$A(x) = \begin{cases} \frac{7}{26}x^2 & \text{if } 0 \le x \le 13, \\ 73.5 - \frac{7}{16}(x-21)^2 & \text{if } 13 \le x \le 21 \end{cases}$$

2. After imposing a coordinate system, find the equation of the line from Casserole to Needham. Then find the equation of the line that is perpendicular to this line, and passes through Lakewood. Find the intersection of the two lines, and you'll have the point at which Jessie is closest to Lakewood. Use the distance formula to find the distance from that point to Lakewood, and that's the answer.

   Version 1: 11.3417172... miles.

   Version 2: 10.1353376... miles.

3. Find the equation of the sloping line. Say it is $y = mx + b$. Then the rectangle with width x has height $y = mx + b$, and so has area equal to

   $$A = x(mx + b) = mx^2 + bx$$

   By finding the vertex of this function of x, you can find the width that achieves the maximum area, and the value of the maximum area.

   Version 1: 52.25

   Version 2: 55.25

4. After setting up a coordinate system (ideally with the center of the forest at the origin, though other arrangements would also work), find the equation of the line representing her path, and the equation of the circle representing the boundary of the forest. Find the intersections of the line and the circle. This will tell you where she enters and where she exits the forest. Use the distance formula to find the distance between these points to get the distance she will walk through the forest.

   Version 1: 9.5578 miles

   Version 2: 7.6626 miles