

Precalculus, Math120U  
Midterm II, 5/12/2004

Name: \_\_\_\_\_ Student Number: \_\_\_\_\_

- A non-graphical scientific calculator and one page notes are allowed. Closed-book exam. 50 minutes long.
- Indicate your answer clearly or mark them.
- Always leave your final answer as an exact form. If asked to have an approximated numerical answer, please rounded up to 2 decimals.
- Demonstrate detailed reasonings of your idea for possible partial credits.

Good luck!

1.

2.

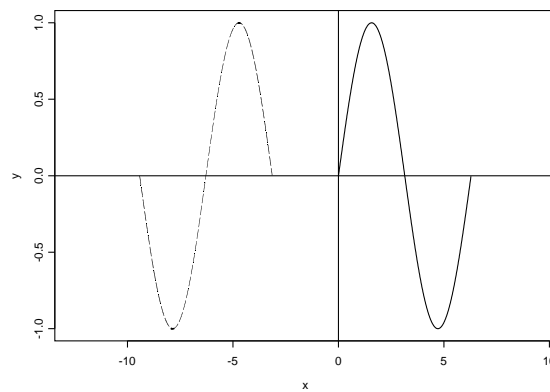
3.

4.

Total.

1. (a) [4 pts] Give 3 different values for  $\theta$  in radian system such that  $\cos(\theta) = -\frac{\sqrt{2}}{2}$ .

(b) [7 pts] The curve of the dash line below is obtained by using combinations of shifting and reflecting on the curve of the solid line, which is the graph of  $y = \sin(x)$  for  $0 \leq x \leq 2\pi$ . Describe a sequence of such geometric maneuvers by words. Then write down the final function whose graph is the curve of the dash line. You don't need to specify the domain and the range.



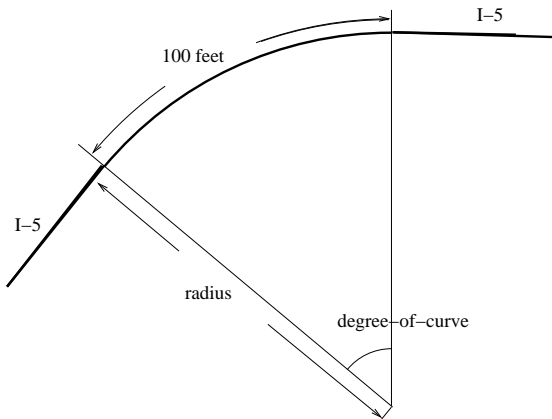
The function that corresponds to the dash curve:  $y =$  \_\_\_\_\_

Descriptions of geometric maneuvers in detail:

2. [13 pts] Show your steps for full credits.

(a) [7 pts] Given a one-to-one function  $y = g(x) = -x^2 + 5$  for  $x \geq 2$ , find the inverse function  $y = g^{-1}(x)$  with its domain and range.

(b) [6 pts] In the highway design, the “degree-of-curve” is the number of degrees created by a 100 feet long arc length. In order to provide safe travel, for a special part of I-5, engineers decide to build a 100 feet long arc with the radius of 5000 feet. Find the degree-of-curve for this part of I-5. Please approximate your answer with 2 decimals.



3. [14 pts] Given a rational function  $y = f(x) = \frac{2x + 3}{x - 5}$ ,

(a) [1 pts] Find the natural domain of  $y = f(x)$ .

(b) [5 pts] Find the  $x$ -intercept and  $y$ -intercept of  $y = f(x)$ .

(c) [5 pts] Assume this rational function is one-to-one, find the inverse function  $y = f^{-1}(x)$ .

(d) [3 pts] Find the horizontal asymptotic line(s) of  $y = f^{-1}(x)$ . Please write your line equation in a proper form.

(e) *Bonus*[3 pts]: Can you write down the range of  $y = f(x)$  without knowing the graph of  $y = f(x)$ ?

4. [12 pts] For this problem, we're concerned with the linear speed for different spots on the earth. Assume the earth is a perfect sphere, it is known that the earth rotates at a linear speed of 1700 kilometers per hour at the Equator.

(a) [3 pts] Use your common sense to find the angular speed of spots on the earth at the Equator. Your answer should be in terms of radian per hour. (You don't need to know anything about the radius of the earth)

(b) [3 pts] Denote the radius of the earth as  $R$ , we know that all spots on the earth at  $60^\circ$  Latitude form a small circle with radius  $r$ . The picture below shows the meaning of Latitude. Express  $r$  in terms of  $R$ .

(c) [6 pts] Find the linear speed of spots on the earth at  $60^\circ$  Latitude. Your answer should be a number with the appropriate unit and should not contain  $R$ .

