Precalculus, Math120U
Midterm 1, 4/21/2004

Name:———————————————————— **Student Number:**——————

- A non-graphical scientific calculator and one page notes are allowed. Closed-book exam. 50 minutes long.

- Indicate your answer clearly or mark them.

- Always leave your final answer as an exact form or a numerical answer rounded up to 2 decimals if asked.

- Demonstrate detailed reasonings of your idea for possible partial credits.

Good luck!!

1.

2.

3.

4.

Total.

1. [8 pts] Given 4 functions defined on their natural domains,

$$f_1(x) = x, \quad f_2(x) = \sqrt{1 - x^2}, \quad f_3(x) = -1, \quad f_4(x) = 2x^2 - 5$$

(a) Write down the new equations for the following compositions without concerning about domains and ranges:

$f_1(f_1(x)) =$ $f_3(f_2(x)) =$

$f_4(f_2(x)) =$ $f_4(f_3(x)) =$

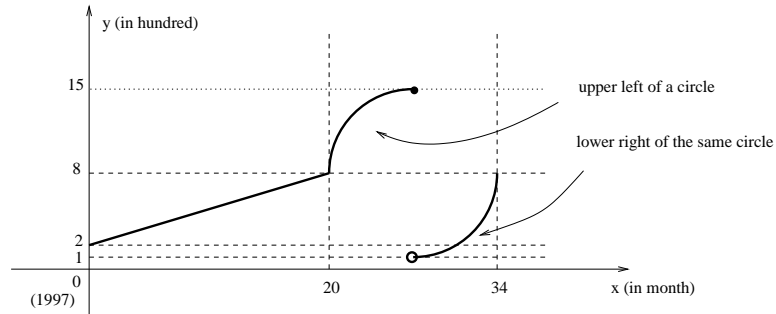(b) Find the largest possible domain for the composition $f_4(f_2(x))$. [4 pts]

2. [10 pts] Given 2 points in a coordinate system, A: $(3, \frac{1}{2})$, B: (0, -1). We want to describe those points such that their distance to point A is the same with their distance to point B.

(a) Let a point Q be (x, y), translate "the distance between Q and A = the distance between Q and B" into a mathematical equation with x and y. [5 pts]

(b) Simplify your equation in (a) as much as possible to the form of $y = f(x)$. [5 pts]

3.[14 pts] A company XYZ was established in Seattle at the beginning of 1997 with 2 hundred employees. Within the first 20 months, it grows with a constant rate to 8 hundred employees. For the following 14 months, the number of employees experienced a non-linear growth to 15 hundred, then a huge layoff and finally a gradual recovery from 1 hundred back to 8 hundred.

If we let x be the number of months from the time the company founded, y the number of employees in hundred, we can have the following graph to show the relationship between x and y.
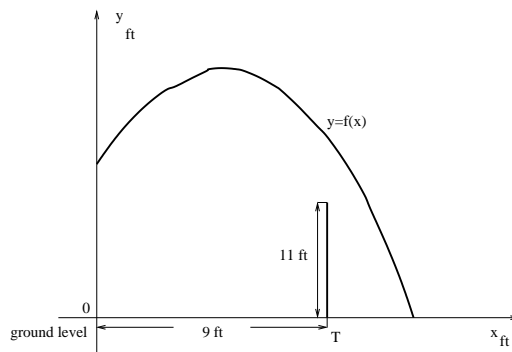

(a) Find that constant rate of change of employees per month within the first 20 months. Don't forget to write down the right unit with your answer. [4 pts]

(b) Write down this multi-part function $y = f(x)$ explicitly. [6 pts]

(c) For how long did the company have more than 10 hundred employees? Write your final answer in an exact form or approximate it by 2 decimals. [4 pts]

4.[18 pts] A stone is moving along the path that follows the graph of $y = f(x) = -x^2 + 7x + 30$.

(a) Please complete the square of $y = f(x)$, that is to rewrite $y = f(x)$ as $A(x + B)^2 + C$ for some constant A, B, C. [5 pts]

(b) What is the maximum height that the stone can reach? [3 pts]

(c) What are the coordinates of the spot where the stone hits the ground? [5 pts]

(d) If at the spot T on the picture, there is a 11-ft high tree. Will the stone hit the tree? Explain your answer mathematically. (the tree may not be drawn proportionally on the picture) [5 pts]

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